

Nuclear Effects in Electron & Neutrino DIS

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Outline

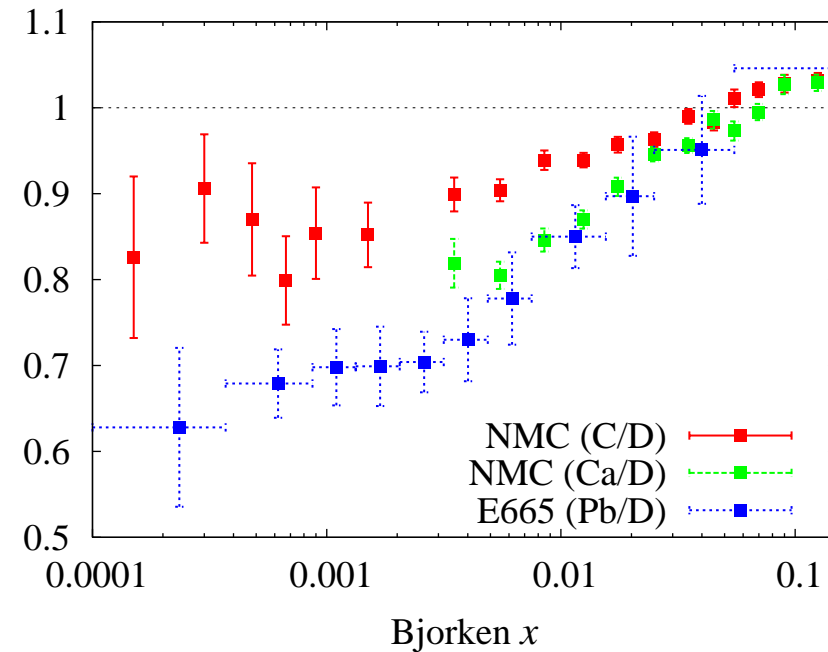
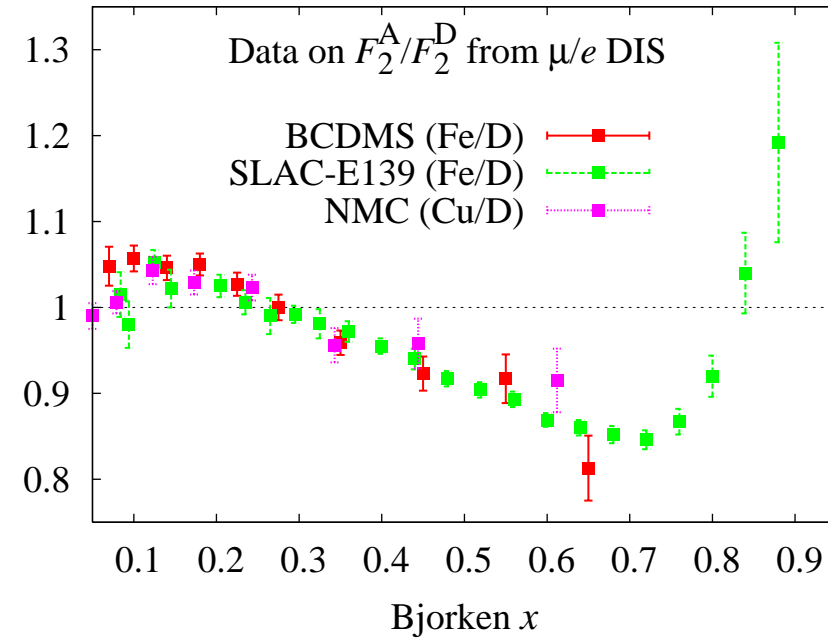
- Experimental evidence of nuclear effects in parton distributions/structure functions. (charged-lepton DIS and hadronic Drell–Yan reactions).
- The development of a realistic model of nuclear structure functions. Phenomenology of nuclear DIS.
- Results for neutrino-nuclear structure functions.

Experimental evidence of nuclear effects in DIS

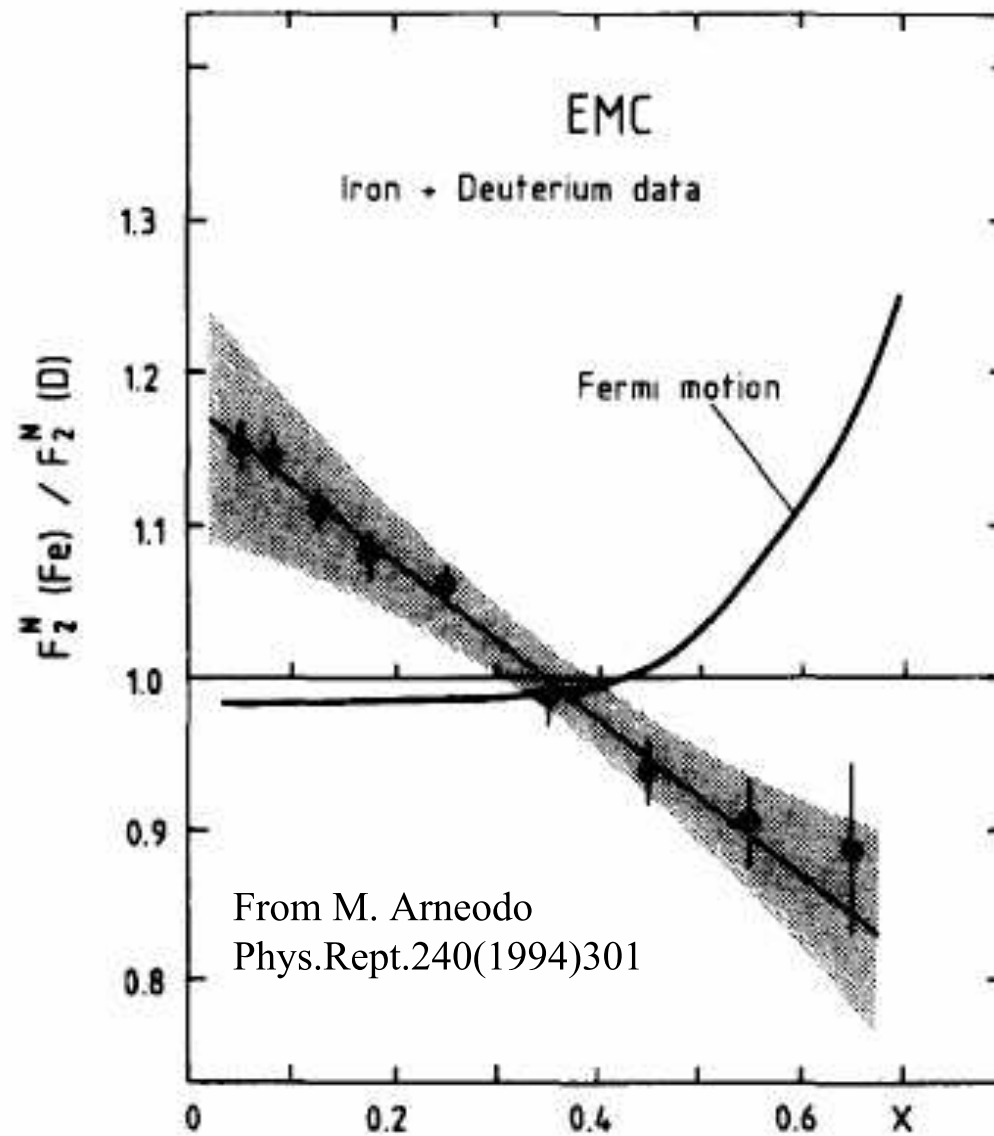
- Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}_2(A/B) = F_2^A/F_2^B$.
- Targets:
Variety of nuclear targets from ^2D to ^{208}Pb
- Experiments:
 - Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
 - Electron beam at SLAC (E139, E140), HERA (HERMES) and recently at JLab.
- Kinematics and statistics:
Data covers the region $10^{-4} < x < 0.9$ and $0 < Q^2 < 150 \text{ GeV}^2$. About 600 data points with $Q^2 > 1 \text{ GeV}^2$.

Data on the EMC ratios show a weak Q^2 dependence that suggests scaling origin of (at least a part of) nuclear effects. Characteristic nuclear effects were observed for different kinematical regions of the Bjorken x .

- Nuclear shadowing at small values of x ($x < 0.05$).
- Antishadowing at $0.1 < x < 0.25$.
- A well with a minimum at $x \sim 0.6 \div 0.75$ (EMC effect).
- Enhancement at large $x > 0.75 \div 0.8$ (Fermi motion region).

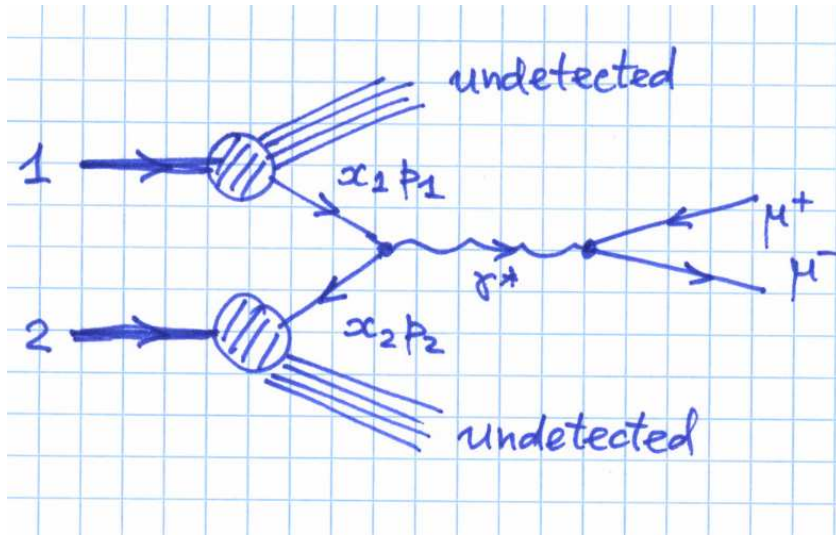


Original observation by European Muon Collaboration



Drell-Yan reaction

Drell-Yan production of lepton pairs in hadron collisions $B + T \rightarrow \mu^+ \mu^- + \text{anything}$



$$\frac{d^2\sigma}{dx_B dx_T} = \frac{4\pi\alpha^2}{9Q^2} K \sum_a e_a^2 \left[q_a^B(x_B) \bar{q}_a^T(x_T) + \bar{q}_a^B(x_B) q_a^T(x_T) \right]$$

$$x_T x_B = Q^2 / s,$$

$$x_B - x_T = 2q_L / \sqrt{s} = x_F$$

Q^2 is invariant mass squared of the lepton pair, q_L the longitudinal momentum of the lepton pair, s the center-of-mass energy squared. Selecting small Q^2/s and large x_F we probe the target's sea.

E772 data on the ratio of Drell–Yan yields

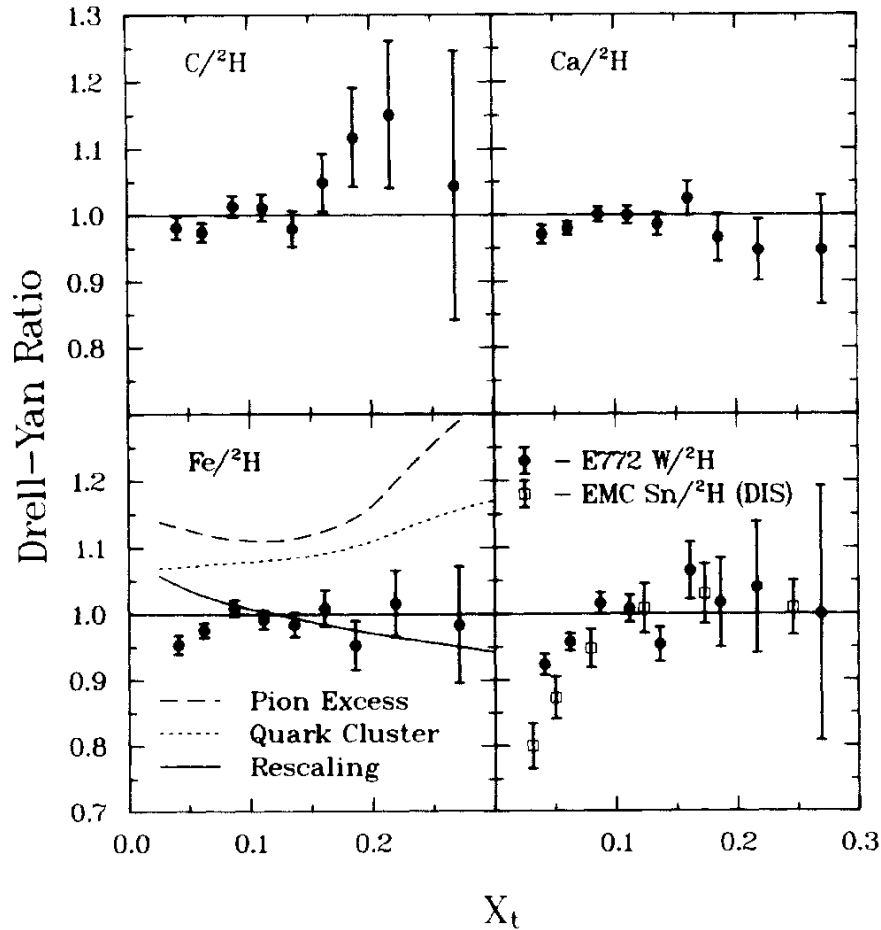


FIG. 3. Ratios of the Drell-Yan dimuon yield per nucleon, Y_A/Y_{2H} , for positive x_F . The curves shown for $Fe/2H$ are predictions of various models of the EMC effect. Also shown are the DIS data for $Sn/2H$ from the EMC (Ref. 4).

In E772 experiment $s = 1600 \text{ GeV}^2$. At $x_F = x_B - x_T > 0.2$ the process is dominated by $q^B \bar{q}^T$ annihilation. The ratio of DY yields for different targets is proportional to the ratio of antiquark distributions:

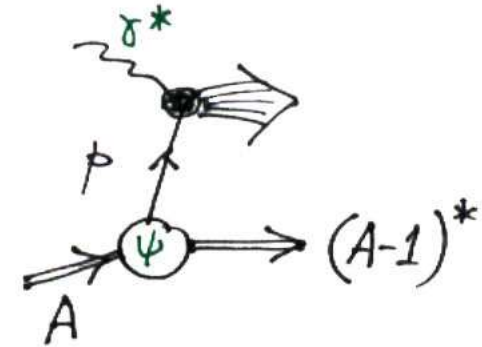
$$\frac{\sigma_A^{\text{DY}}}{\sigma_B^{\text{DY}}} \approx \frac{\bar{q}_A(x_T)}{\bar{q}_B(x_T)}$$

Description of nuclear DIS in Impulse approximation

Fermi motion and nuclear binding corrections (FMB)

$$F_2^A(x, Q^2) = \int d^4p \mathcal{P}_A(p) \left(1 + \frac{p_z}{M}\right) F_2^N(x', Q^2, p^2),$$

$$x = \frac{Q^2}{2p \cdot q}, \quad x' = \frac{Q^2}{2p \cdot q} = \frac{x}{1 + (\varepsilon + k_z)/M}$$

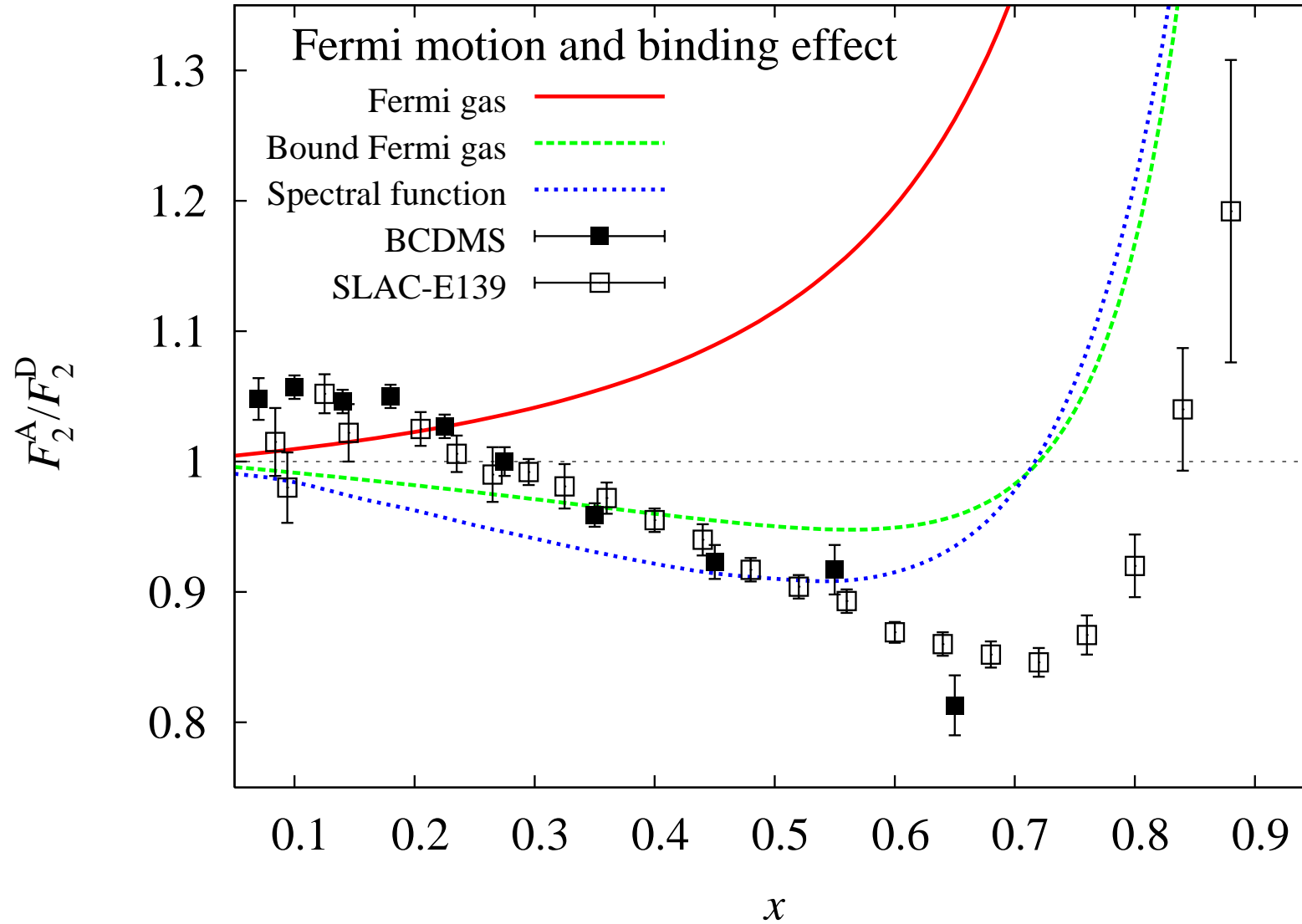


Similar equations hold in impulse approx. for other structure functions (F_T , F_3). Fermi motion and binding effect is driven by nuclear spectral function

$$\mathcal{P}_A(p) = \sum_n |\psi_n(\mathbf{p})|^2 \delta(\varepsilon + E_n(A-1, -\mathbf{p}) - E_0(A)).$$

Spectral function describes probability to find a bound nucleon with momentum \mathbf{p} and energy $p_0 = M + \varepsilon$.

EMC Ratio and FMB correction



Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M + \varepsilon)^2 - \mathbf{p}^2 < M^2$. In off-shell region nucleon structure functions depend on additional variable p^2 .

The virtuality parameter $v = (p^2 - M^2)/M^2$ is small on average (e.g. for ^{56}Fe $\langle v \rangle \sim -0.15$). Hence a linear approximation in v

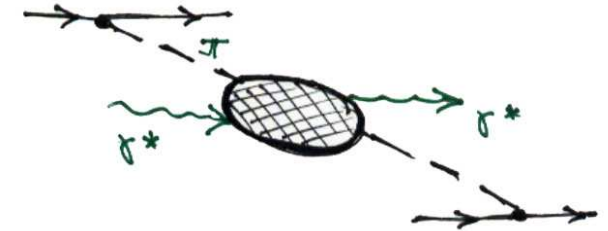
$$F_2^N(x, Q^2, p^2) = F_2^N(x, Q^2) [1 + v \delta f(x)]$$

We follow a phenomenological approach and extract the off-shell function $\delta f(x)$ from data on the nuclear structure functions [S.K. & R.Petti, NPA765(2006)126].

Nuclear pion effect

Leptons can interact with nuclear meson field which mediate interaction between bound nucleons. This process generate a pion correction to nuclear structure functions.

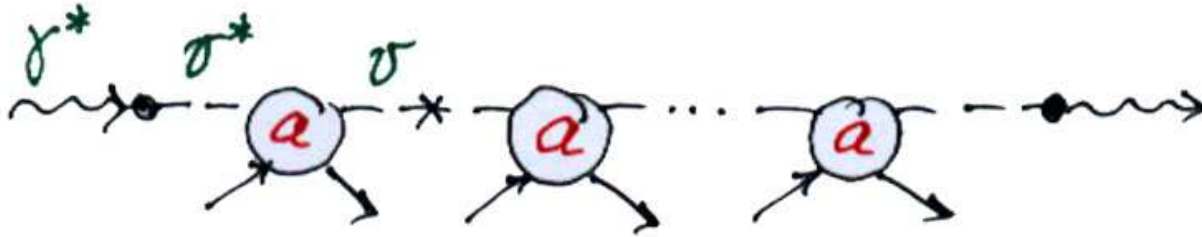
$$\delta F_i^{\pi/A}(x, Q^2) = \int_x dy f_{\pi/A}(y) F_i^{\pi}(x/y, Q^2)$$



- Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y \rangle_{\pi} + \langle y \rangle_N = 1$.
- The nuclear pion distribution function is localized in a region of $y < p_F/M \sim 0.3$. For this reason the pion correction to nuclear (anti)quark distributions is localized at $x < 0.3$.
- The magnitude of the correction is driven by average number of “pions” $n_{\pi} = \int dy f_{\pi/A}(y)$. By order of magnitude $n_{\pi}/A \sim 0.1$ for a heavy nucleus like ^{56}Fe .
- Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

Nuclear DIS in coherent regime: shadowing

At small x DIS is driven by $\gamma^* \rightarrow v^*$ conversions into virtual hadronic states. The nuclear correction at small x comes from multiple interactions of hadronic component of virtual photon during the propagation through matter.



The series can be summed up in a compact form in optical approximation (suitable for large A)

$$\delta\mathcal{R} = \frac{\delta_{\text{coh}} F_T}{F_T^N} \approx \frac{\delta_{\text{coh}} \sigma_T}{\sigma_T} = \text{Im} \left(i a^2 \mathcal{C}_2^A(a) \right) / \text{Im } a,$$

$$\mathcal{C}_2^A(a) = \int_{z_1 < z_2} d^2\mathbf{b} dz_1 dz_2 \rho_A(\mathbf{b}, z_1) \rho_A(\mathbf{b}, z_2) \exp \left[i \int_{z_1}^{z_2} dz' (a \rho_A(\mathbf{b}, z') - k_L) \right].$$

$a = \sigma(i + \alpha)/2$ is (effective) scattering amplitude in forward direction ($\alpha = \text{Re } a / \text{Im } a$), $k_L = Mx(1 + m_v^2/Q^2)$ is longitudinal momentum transfer in the process $v^* \rightarrow v$, which accounts for finite life time of intermediate hadronic state. The presence of k_L suppresses coherent nuclear effect at large x .

Nuclear shadowing

Dependence on C -parity and isospin

The amplitude a_h^I describes interaction of hadronic component of the virtual boson with a nucleon. This amplitude is characterized by helicity state h of the boson ($h = \pm 1$ for transverse polarization and $h = 0$ for longitudinal polariz.) and by isospin I (amplitude is different for proton and neutron).

Average transverse amplitude $a_T^I = (a_{+1}^I + a_{-1}^I)/2$

Helicity asymmetry $a_\Delta^I = (a_{+1}^I - a_{-1}^I)/2$

Correspondence with structure functions:

$$a_T^0 \rightarrow F_T^{\nu+\bar{\nu}}$$

$$a_T^1 \rightarrow F_T^{\nu-\bar{\nu}}$$

$$a_\Delta^0 \rightarrow F_3^{\nu+\bar{\nu}}$$

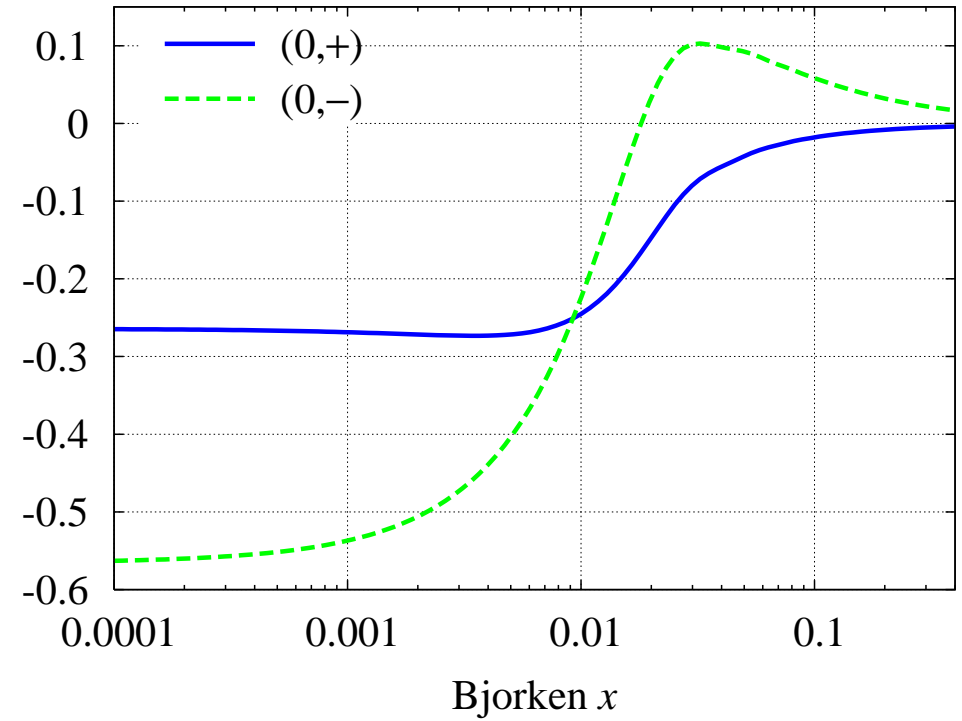
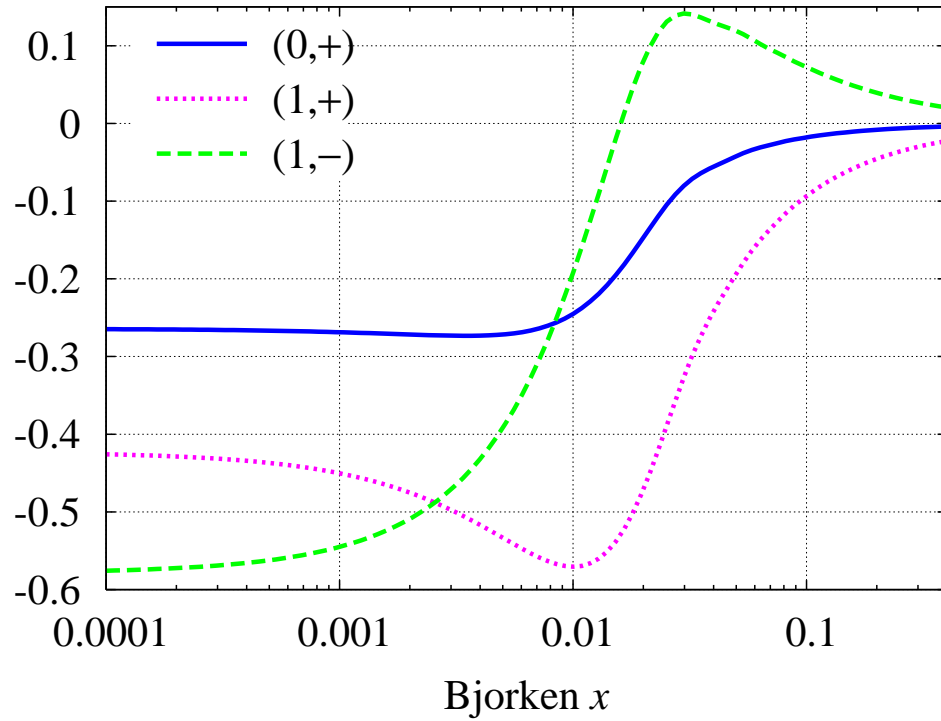
$$a_\Delta^1 \rightarrow F_3^{\nu-\bar{\nu}}$$

Coherent nuclear effects are not universal but depend on the type of the structure function. Nuclear corrections to C -even F_T (or $q + \bar{q}$) and C -odd F_3 (or $q - \bar{q}$) are given by

$$\delta\mathcal{R}^{(0,+)} = \text{Im} \left[i a_T^2 \mathcal{C}_2^A(a_T) \right] / \text{Im} a_T,$$

$$\delta\mathcal{R}^{(0,-)} = \text{Im} \left[i a_\Delta \frac{\partial}{\partial a_T} \left(a_T^2 \mathcal{C}_2^A(a_T) \right) \right] / \text{Im} a_\Delta = \text{Im} \left[i(i + \alpha_\Delta) \frac{\partial}{\partial a_T} (\dots) \right]$$

Note that C -odd correction $\delta\mathcal{R}^{(0,-)}$ depends on $\alpha_\Delta = \text{Re} a_\Delta / \text{Im} a_\Delta$ but not on the corresponding cross section and the strength of nuclear corrections is driven by σ_T , the cross section in the C -even channel.



The ratio $\delta\mathcal{R}^{(I,C)}$ calculated for different isospin and C -parity scattering states for ^{207}Pb at $Q^2 = 1 \text{ GeV}^2$. The labels on the curves mark the values of the isospin I and C -parity, (I, C) .

Phenomenology of nuclear DIS

The idea and motivation:

- Take into account major nuclear corrections and build a quantitative model for nuclear structure functions (for more detail see S.K. & R.Petti, NPA765(2006)126)

$$F_i^A = F_i^{p/A} + F_i^{n/A} + \delta_\pi F_i^A + \delta_{\text{coh}} F_i^A$$

- Parameterize off-shell correction function $\delta f(x)$ and effective scattering amplitude a_T responsible for nuclear shadowing
- Calculate nuclear structure functions, test with data and extract parameters from data.
- Verify the model by comparing the calculations with data not used in analysis as well as with the constraints due to normalizations and the sum rules.

Hadronic/nuclear input to the model

- Proton and neutron SFs computed in NNLO pQCD + TMC + HT using phenomenological PDFs and HTs from fits to DIS data (Alekhin).
- Realistic two-component nuclear spectral function (mean-field + correlated part) is used to calculate SF in impulse approximation.
- Mesonic correction is calculated using pion PDFs extracted from fits to Drell–Yan data.
- Coherent nuclear corrections are calculated using multiple scattering theory and nuclear number densities $\rho_A(\mathbf{r})$ from elastic electron scattering data.

Analysis

We use the data from electron and muon DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variety of targets. The data are available for A/D and $A/^{12}\text{C}$ ratios.

We perform a fit to minimize $\chi^2 = \sum_{\text{data}} (\mathcal{R}_2^{\text{exp}} - \mathcal{R}_2^{\text{th}})^2 / \sigma^2(\mathcal{R}_2^{\text{exp}})$ with σ the experimental uncertainty. In the fit we use data with $Q^2 > 1 \text{ GeV}^2$ (overall about 560 points). Then we validate the predictions for $Q^2 < 1 \text{ GeV}^2$.

We use the parametrization $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$ for the off-shell function. For the effective scattering amplitude of hadronic component of γ^* off the nucleon we use the following model

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2,$$

$$\bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

Preliminary trials: Initially the parameters σ_1 , σ_0 , α_T , Q_0^2 and x_1 , x_0 , h , C_N are free and we study the correlations between them.

- * The parameter h turned out fully correlated with x_0 , $h = 1 + x_0$ fixed in the final fit.
- * Best fit gives $\sigma_1 \approx 0$. The correlations between σ_1 and off-shell parameters are negligible. We fix $\sigma_1 = 0$.
- * The parameter σ_0 is strongly correlated $\sigma_0 \leftrightarrow Q_0^2$, $\sigma_0 \leftrightarrow \alpha_T$. Best fit gives $\alpha_T = -0.179 \pm 0.038$ and $\Delta\chi^2 \sim 29$ compared to the fit with $\alpha_T = 0$. If we fix $\sigma_0 = 27 \text{ mb}$ (average cross section in the VMD model) then $\alpha_T = -0.182 \pm 0.037$. This is in good agreement with results on the analysis of ρ^0 meson production in VMD. In final fit we fix $\sigma_0 = 27 \text{ mb}$ and $\alpha = -0.2$.

The adjustable parameters of the model are x_0 , x_1 , C_N and Q_0^2 . Additional constraint of the fit is the normalization of nuclear valence quark number. This allows us to constrain x_1 . For this purpose we use an iterative algorithm:

1. Fit with fixed x_1 without normalization constraint.
2. Calculate $\delta N_{\text{val}} = \delta N_{\text{val}}^{\text{off-shell}} + \delta N_{\text{val}}^{\text{shadowing}}$ as a function of Q^2 at $Q^2 > 5 \text{ GeV}^2$
3. Change x_1 and go to 1. until $|\delta N_{\text{val}}|$ is minimized provided that χ^2 is still "acceptable".

Off-shell function

The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.

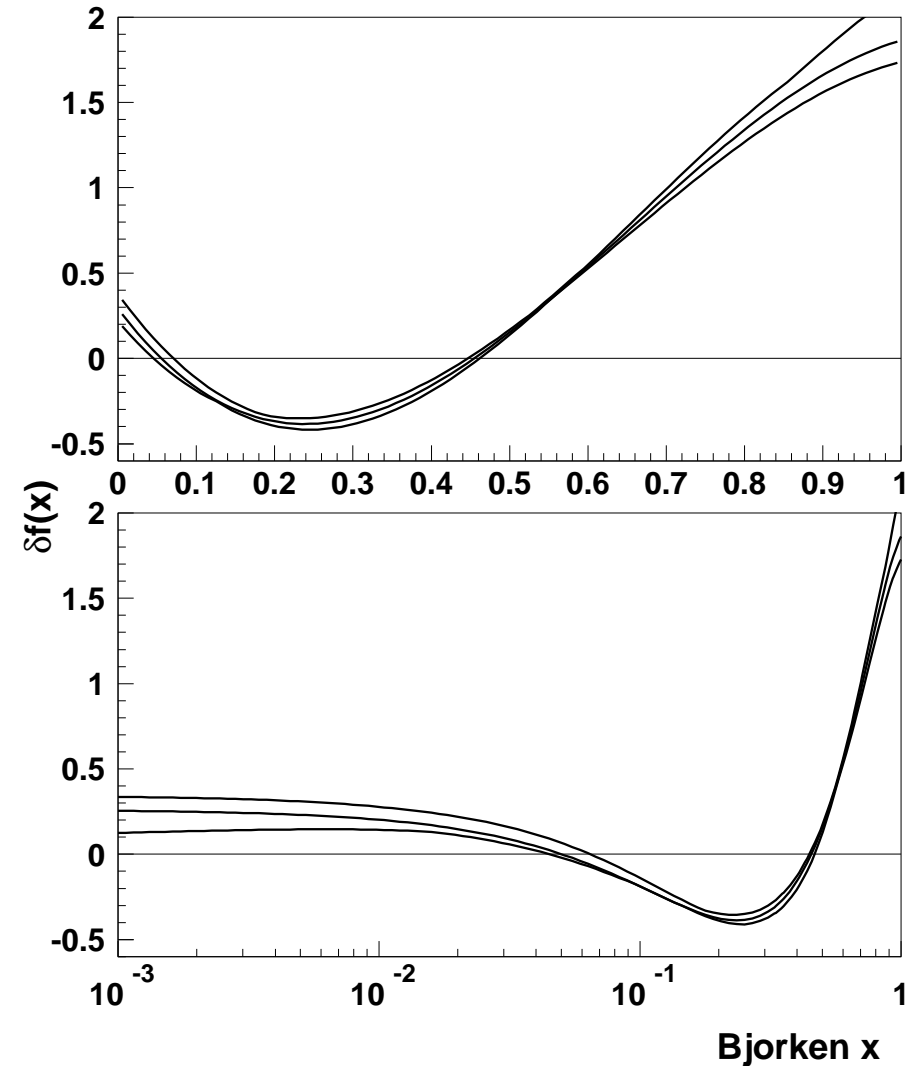
$$\delta f(x) = C_N(x - x_1)(x - x_0)(1 + x_0 - x)$$

$$C_N = 8.1 \pm 0.3 \pm 0.5$$

$$x_0 = 0.448 \pm 0.005 \pm 0.007$$

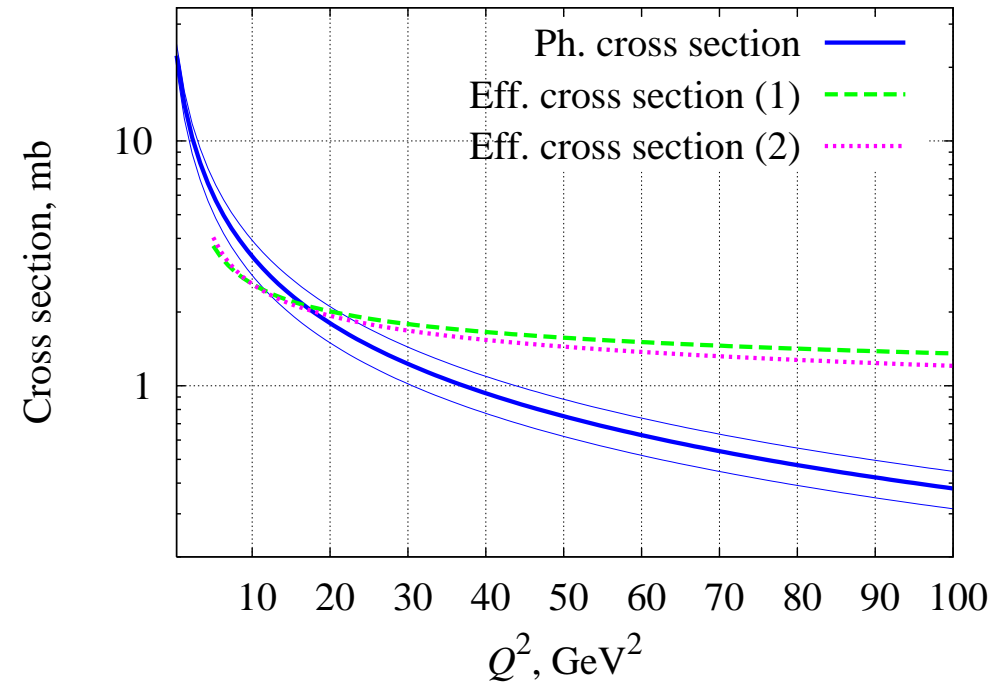
$$x_1 = 0.05$$

- Parameters from the global fit (all nuclei) are consistent with independent fits to different subsets of nuclei
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.

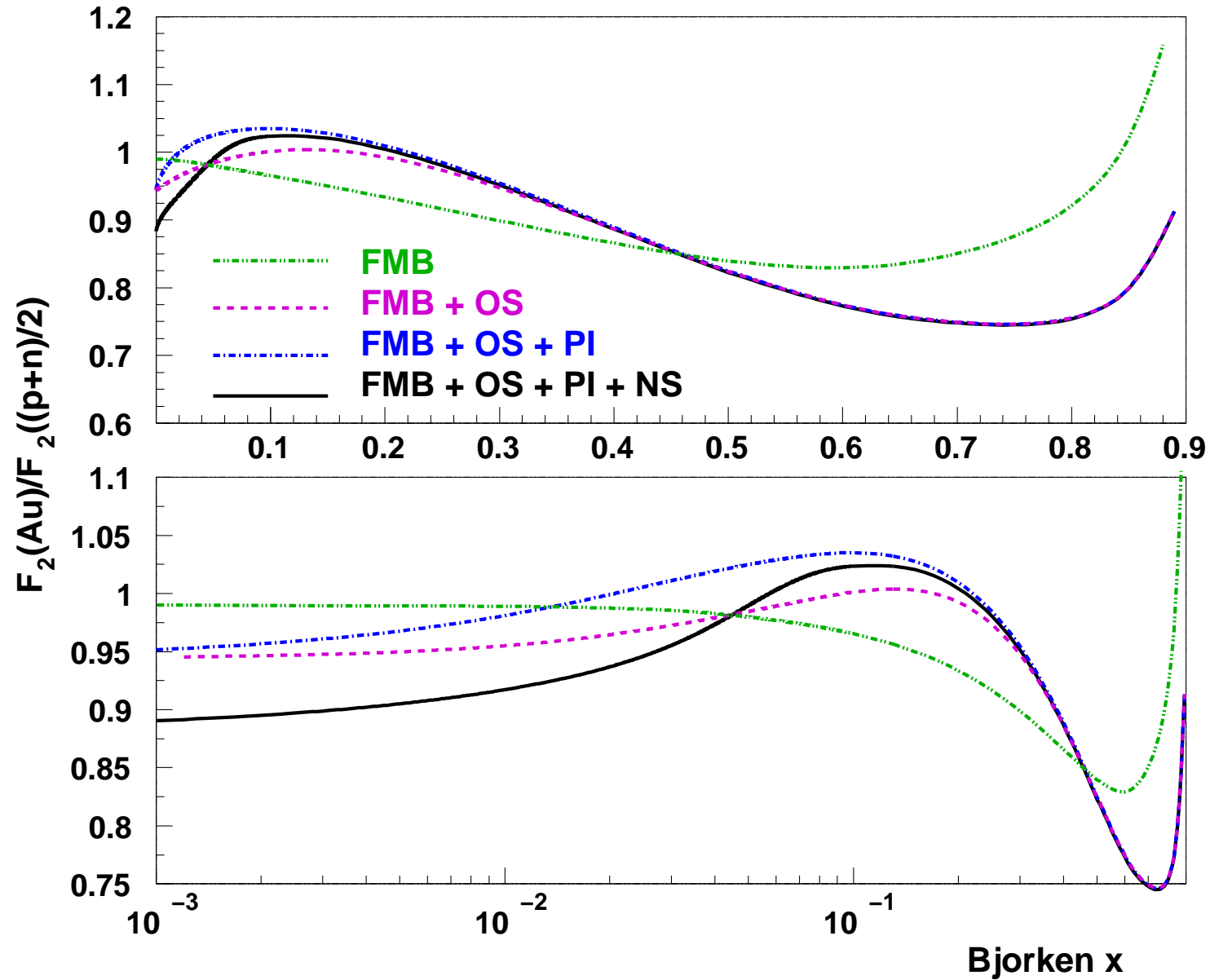


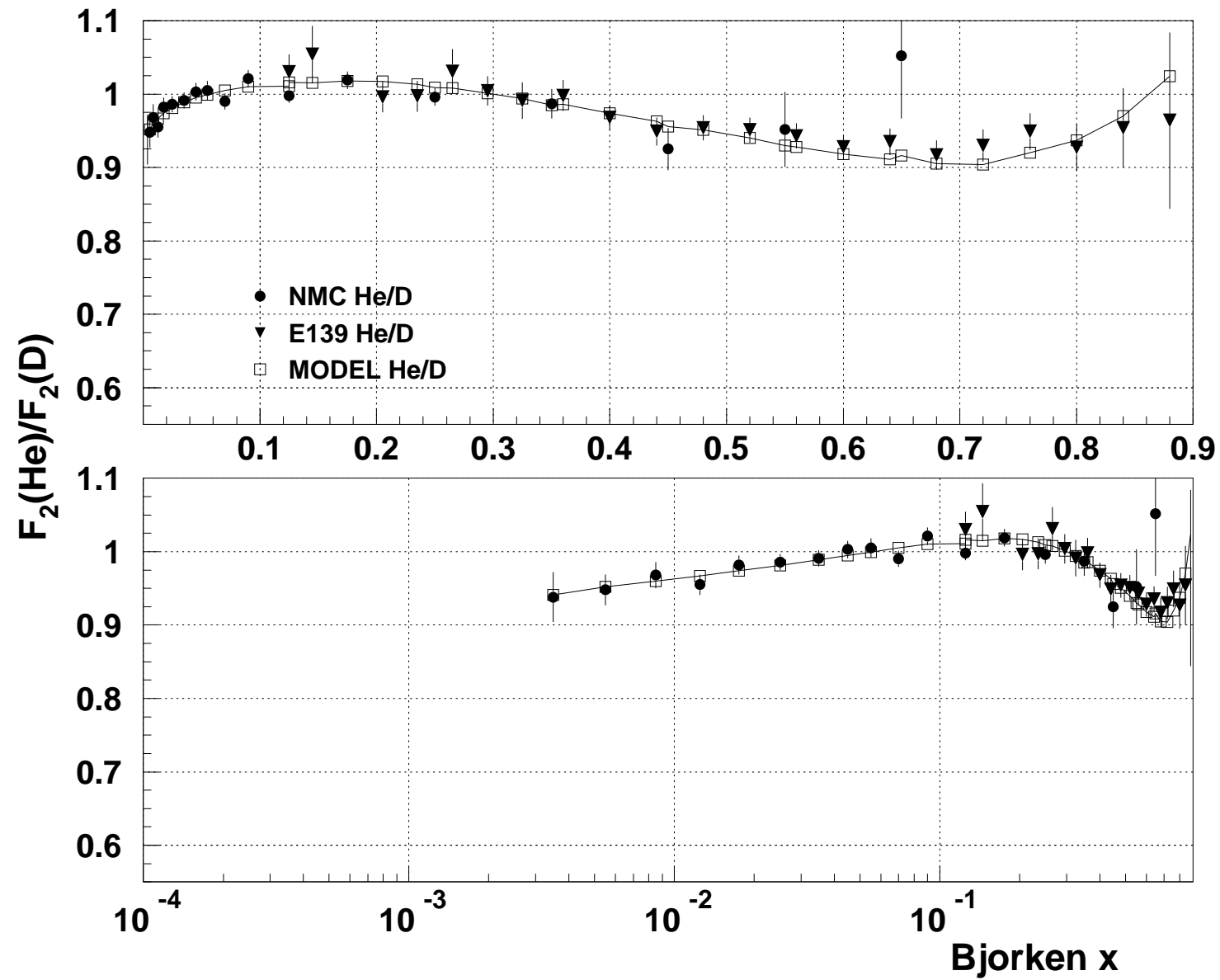
Effective cross section

- The monopole form $\bar{\sigma} = \sigma_0 / (1 + Q^2/Q_0^2)$ provides a good fit to existing DIS data on nuclear shadowing for $Q^2 < 20 \text{ GeV}^2$.
- This does not mean that $\bar{\sigma}$ should vanish as $1/Q^2$ since data are limited to $Q^2 < 20 \text{ GeV}^2$. Effective cross section at high Q^2 can be calculated from the normalization condition of the valence quark distribution $\delta N_{\text{val}}^{\text{off-shell}} + \delta N_{\text{val}}^{\text{shadowing}} = 0$.

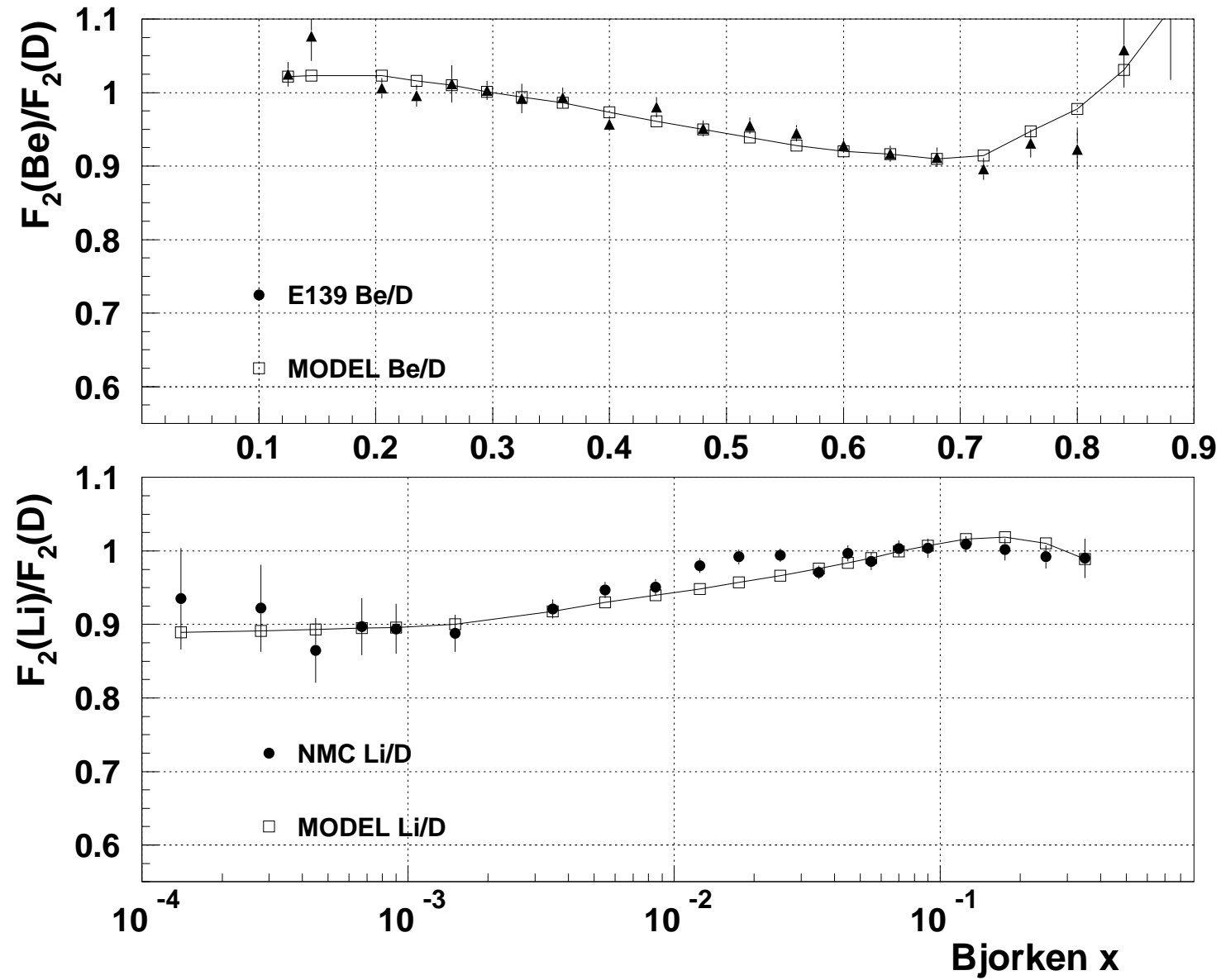


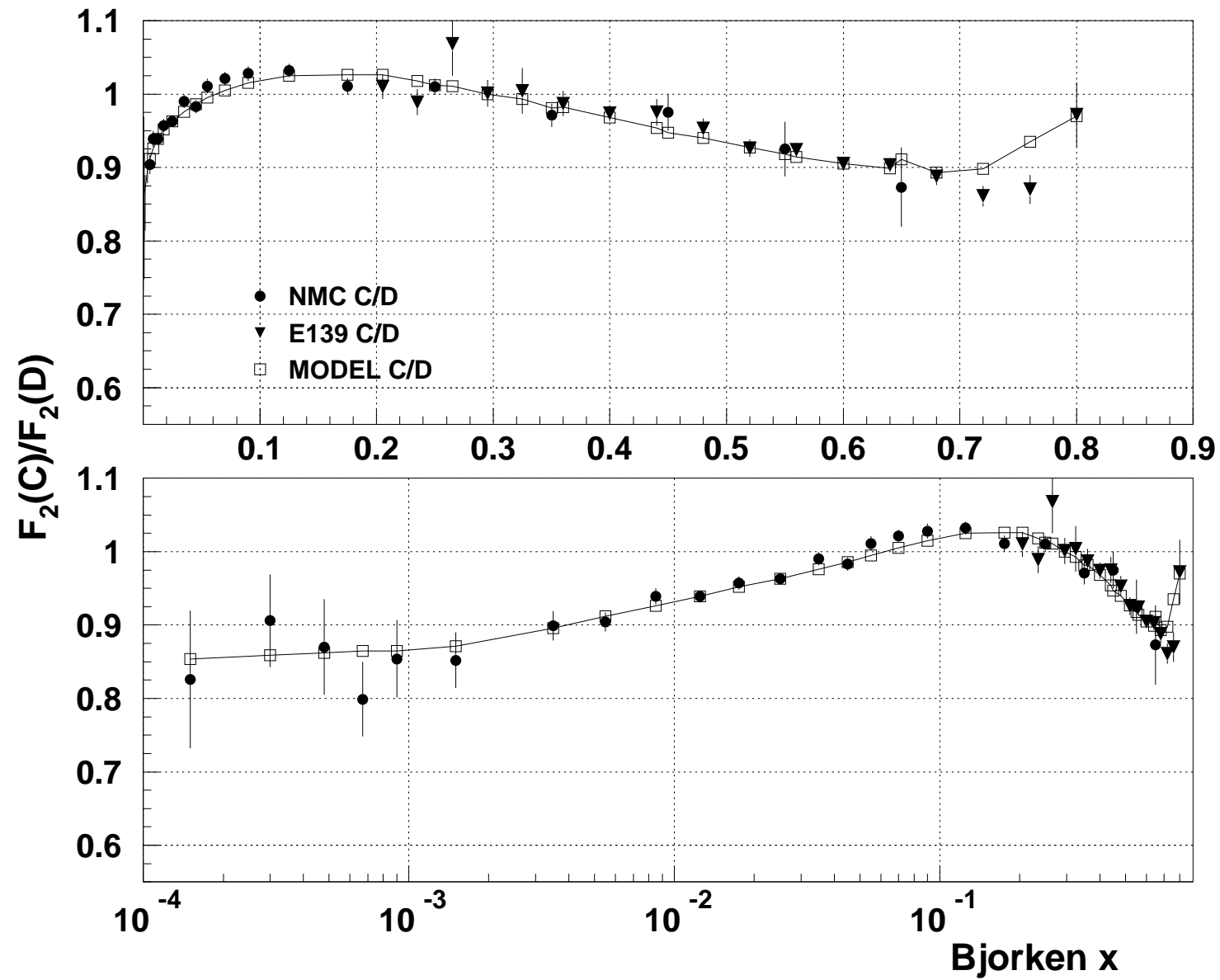
Results for the ratios $\mathcal{R}_2(x, A/B) = F_2^A/F_2^B$

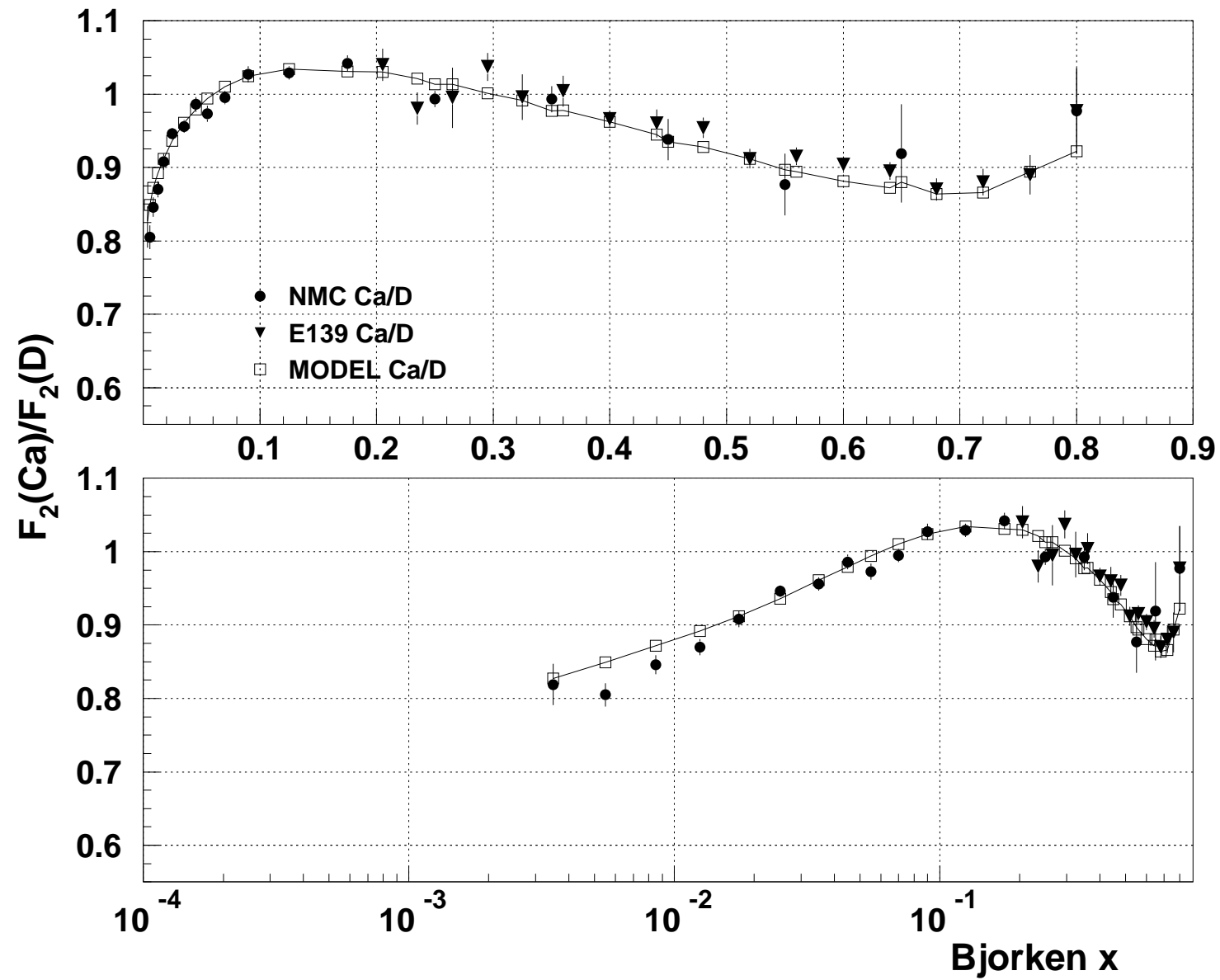


${}^4\text{He}/\text{D}$ 

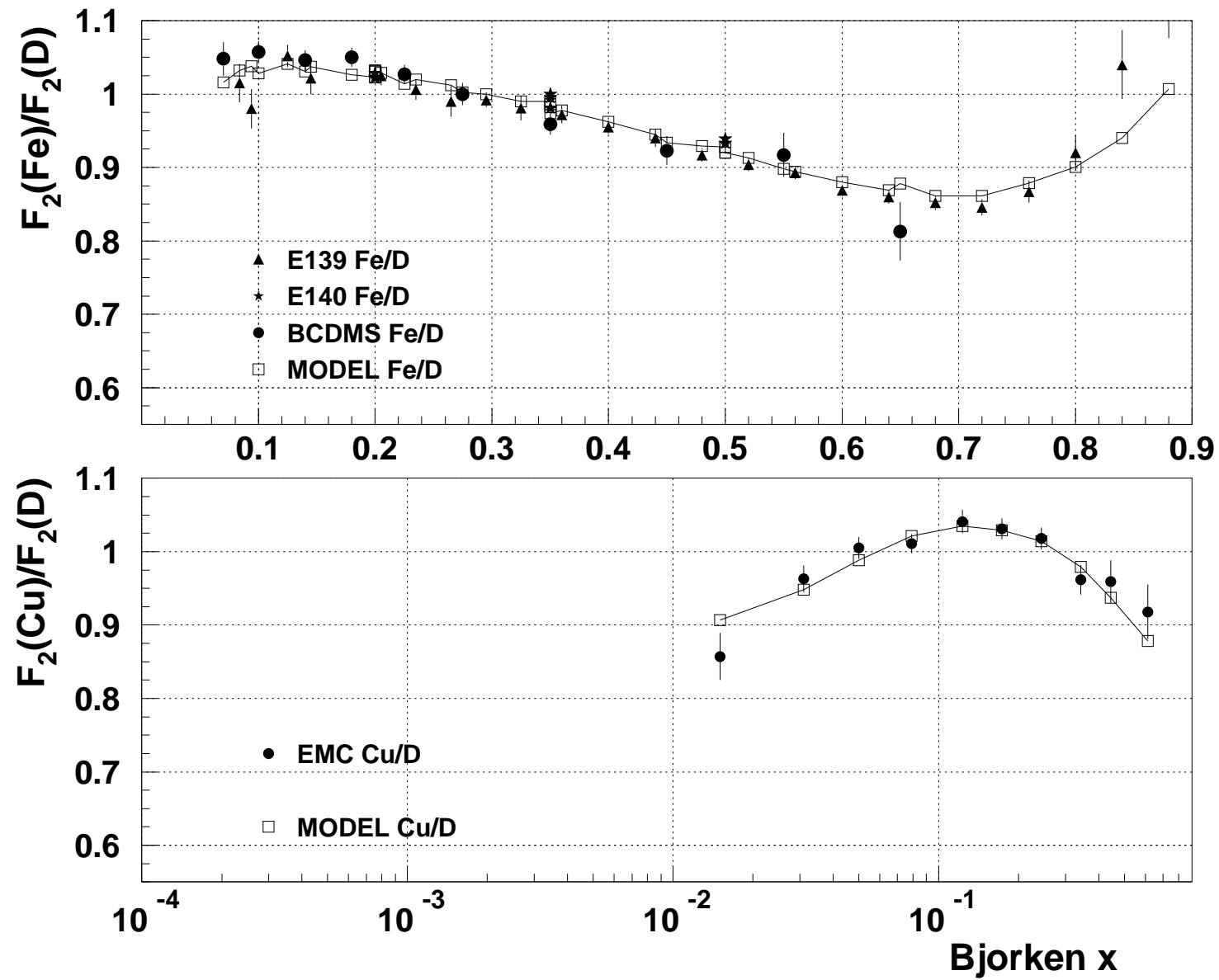
${}^7\text{Li}/\text{D}$ & ${}^9\text{Be}/\text{D}$

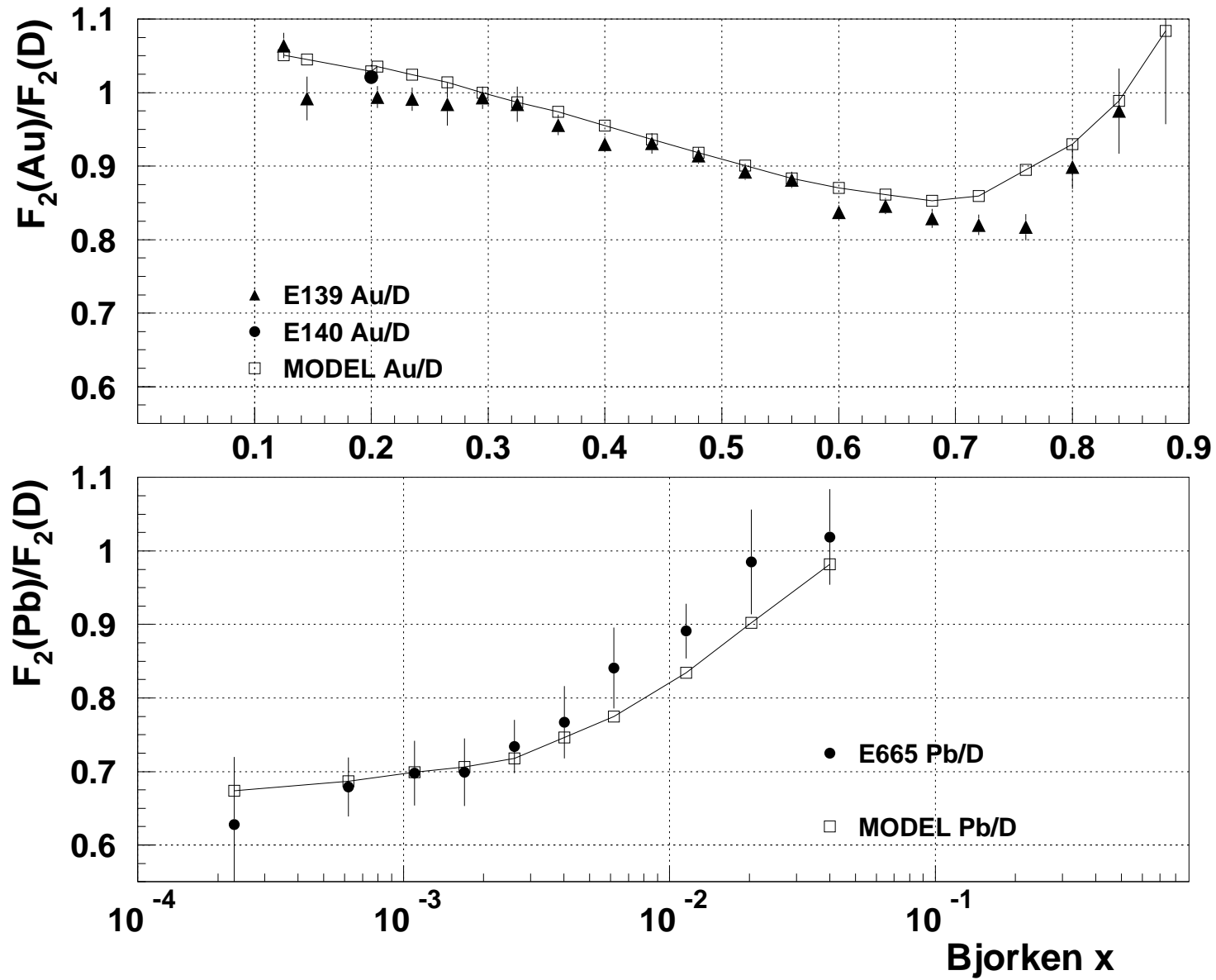


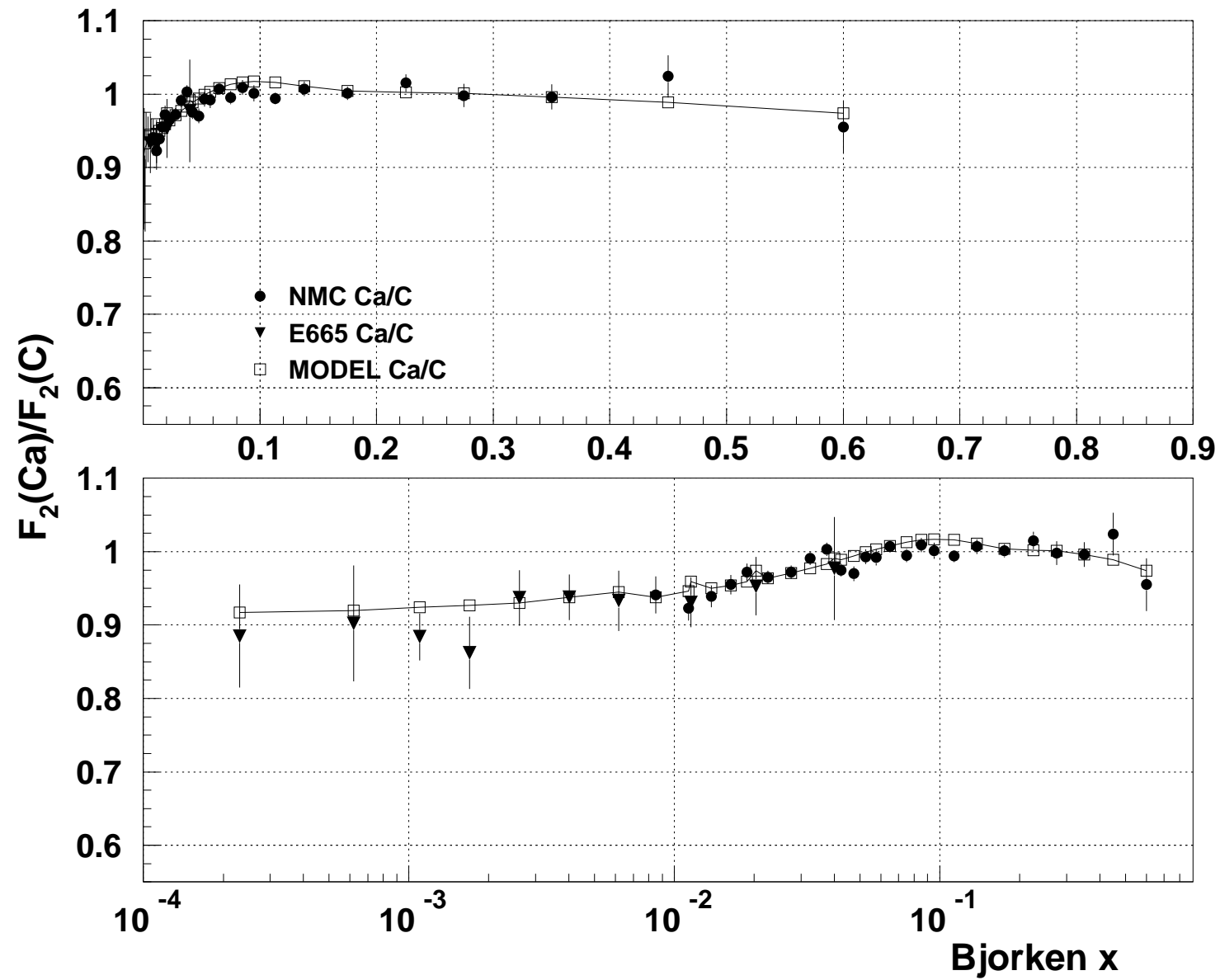
$^{12}\text{C}/\text{D}$ 

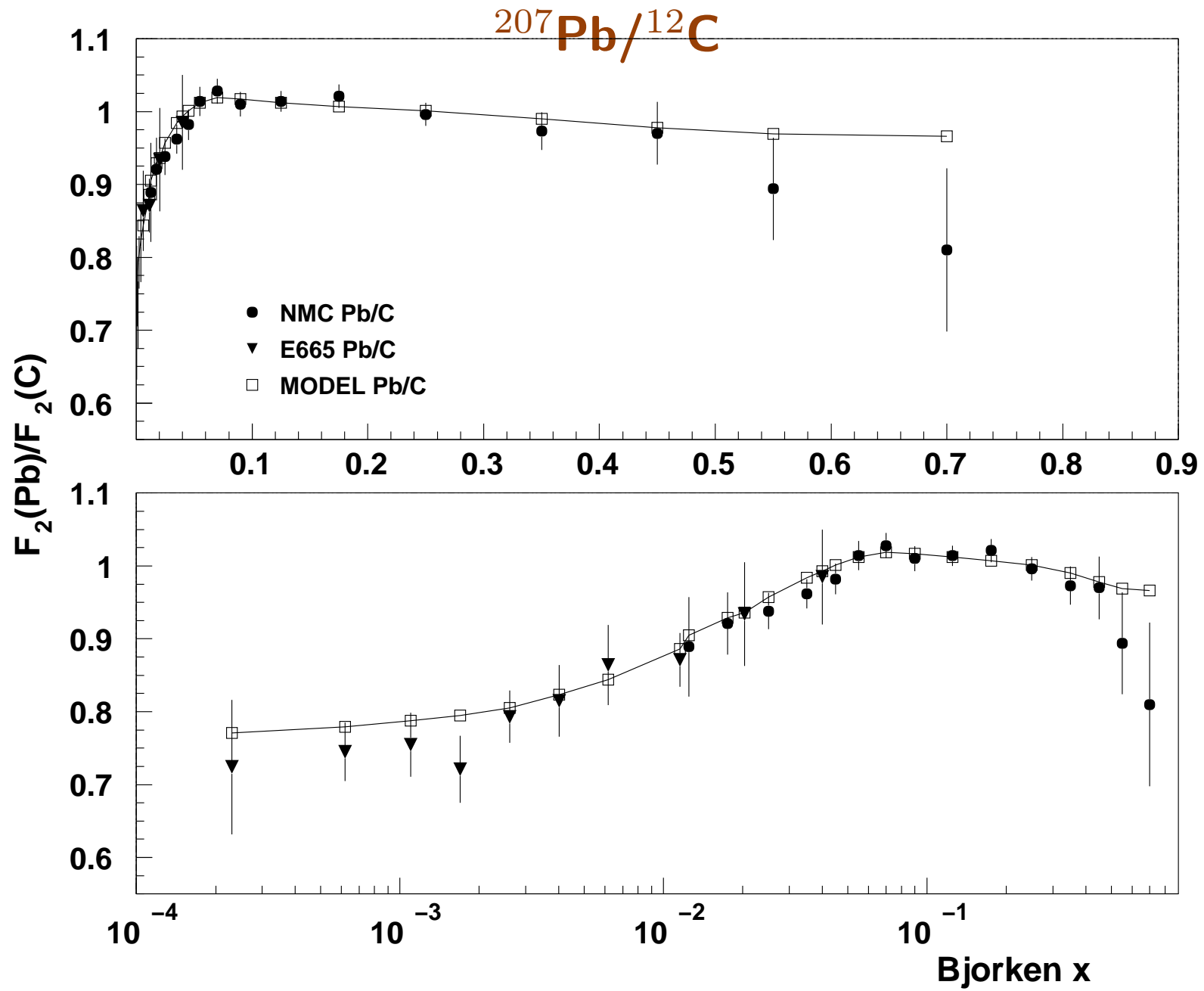
⁴⁰Ca/D

$^{56}\text{Fe}/\text{D}$ & $^{63}\text{Cu}/\text{D}$

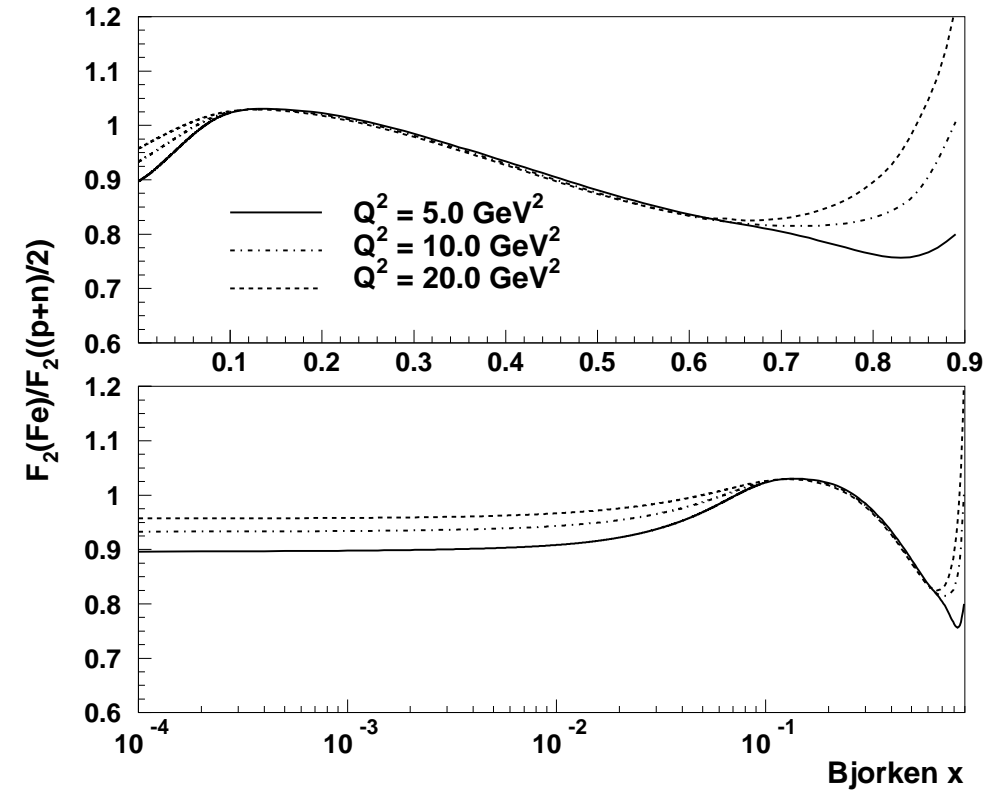
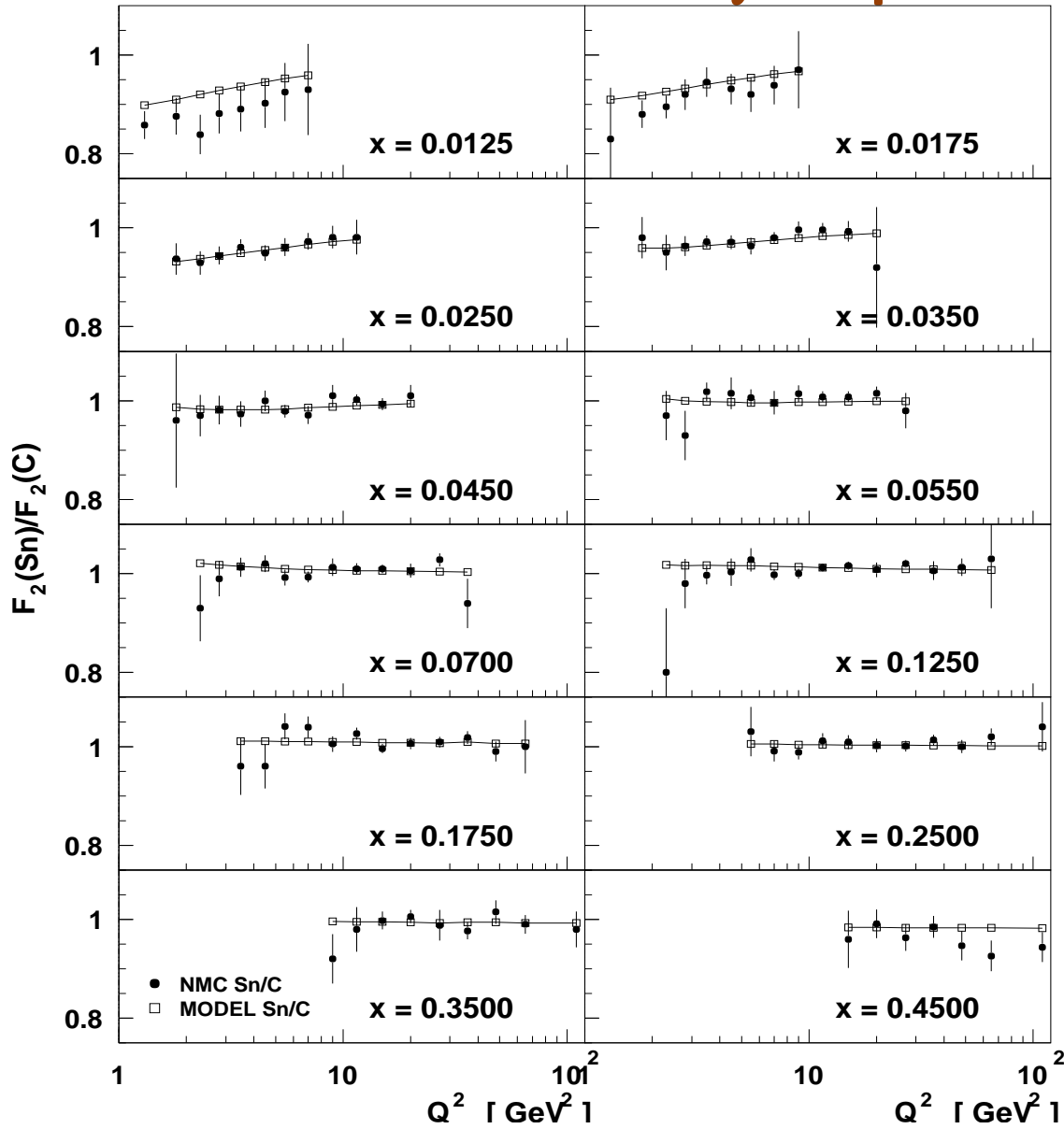


$^{197}\text{Au}/\text{D}$ & $^{207}\text{Pb}/\text{D}$


$^{40}\text{Ca}/^{12}\text{C}$ 



Q^2 dependence of \mathcal{R}_2

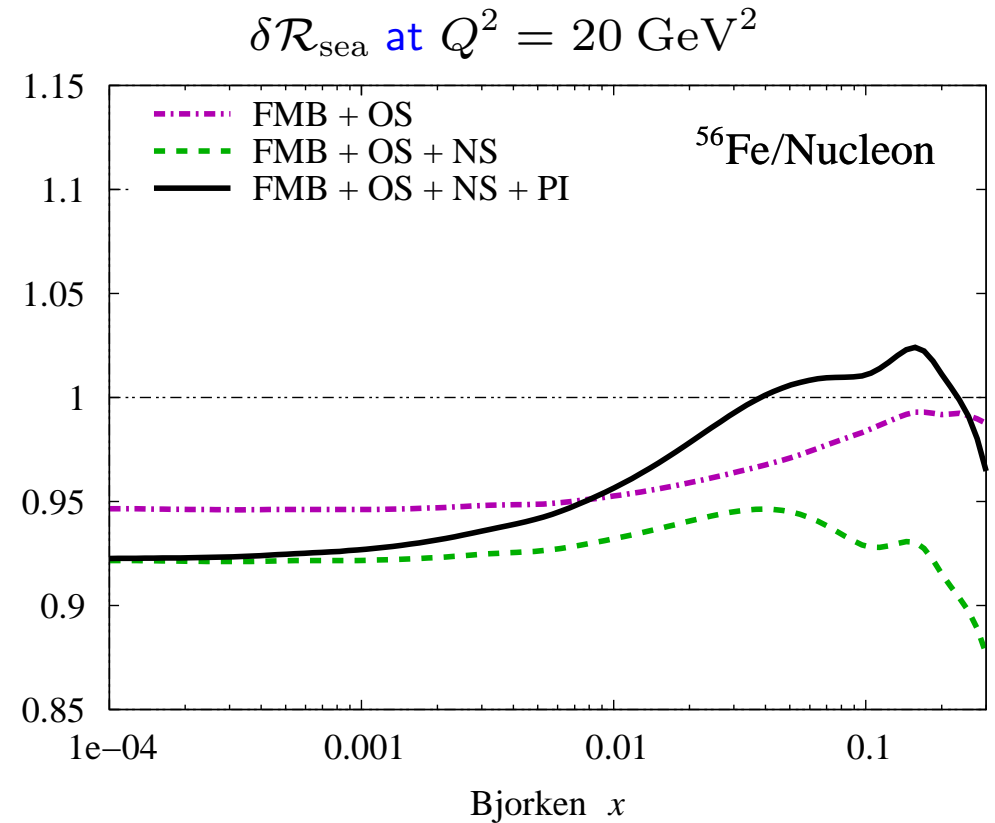
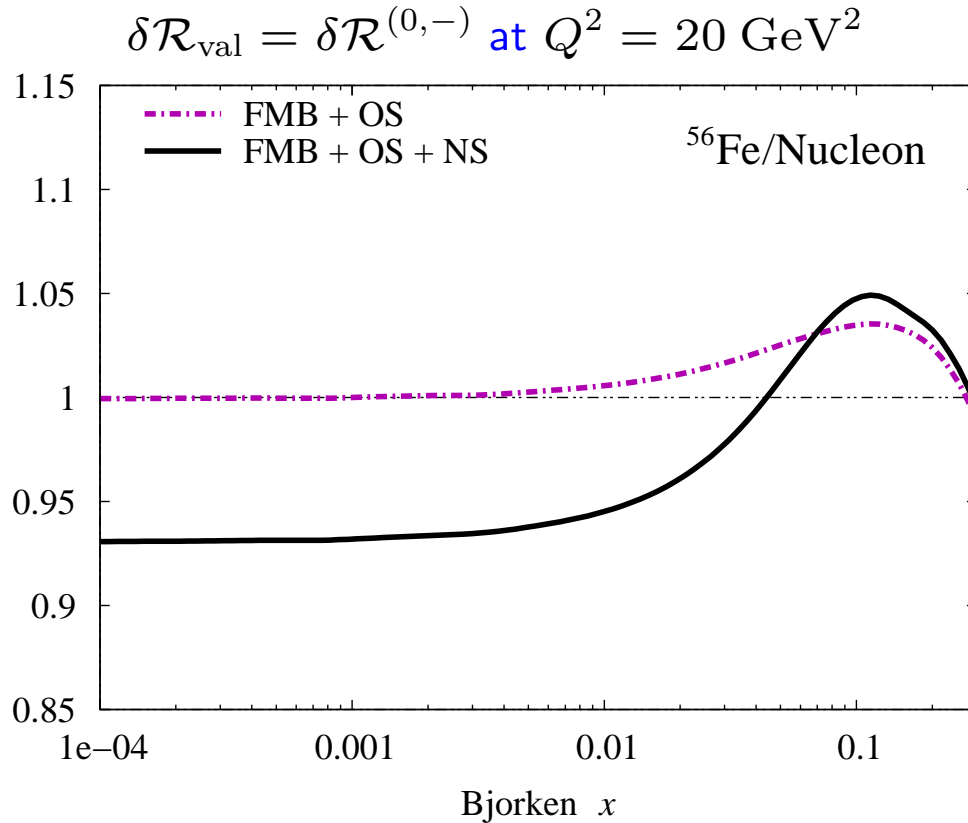


Q^2 dependence of \mathcal{R}_2 was observed for $x < 0.05$ (the Q^2 dependence of shadowing effect) and for $x > 0.7$ (the Q^2 dependence of target mass correction)

Nuclear sea and valence quark distributions

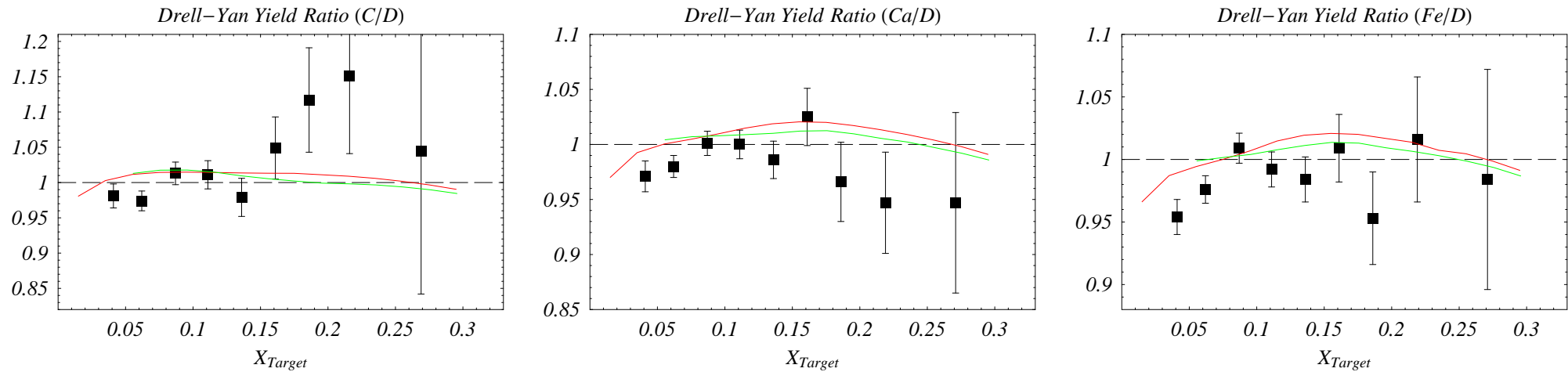
Nuclear corrections for antiquark distribution can be derived from $\delta\mathcal{R}^{(0,-)}$ and $\delta\mathcal{R}^{(0,+)}$:

$$\delta\mathcal{R}_{\text{sea}} = \frac{\delta\bar{q}_A}{\bar{q}_N} = \delta\mathcal{R}^{(0,+)} + \frac{q_{\text{val}/N}(x)}{2\bar{q}_N(x)} \left(\delta\mathcal{R}^{(0,+)} - \delta\mathcal{R}^{(0,-)} \right)$$



⇒ Note remarkable cancellation between pion and shadowing effects for nuclear antiquark distribution for intermediate region of Bjorken x .

Comparison with Drell-Yan data



Calculations are in reasonable agreement with data on DY ratios (Note that the mass of dimuon pair was not given by E772 and calculation was done for a fixed $Q^2 = 20 \text{ GeV}^2$).

The cancellation from shadowing reconciles nuclear pion excess with DY data.

Summary for the CL DIS analysis

- The model is in a very good agreement with data for the entire kinematical region of x and Q^2 and for all studied nuclei from ^4He to ^{207}Pb with overall $\chi^2/\text{d.o.f} = 459/556$ for a 4 parameter fit.
- The fit parameters are common to all nuclei. The parameters are stable for the fits with different subsets of nuclei (see Table 2 of NPA paper).
- The off-shell effect is related to modification of the nucleon structure in nuclear environment. The parameters of $\delta f(x)$ suggest an increase in the size of the bound nucleon valence region (in ^{56}Fe by $\sim 10\%$ on average).
- Not discussed here but worth to mention. Predictions for the deuteron are in a good agreement with E665 D/p data and also with Gomez et.al. extraction of $D/(p+n)$ ratio (except for the region $x > 0.7$).

Nuclear effects in neutrino DIS

- We apply the model developed for charged-lepton nuclear scattering for neutrino-nuclear interactions (for more details see S.K. & R.Petti, PRD46(2007)094023).
- Additional input is required to treat the major nuclear effects for νA scattering.
 - Off-shell corrections for different structure functions (F_2 and F_3) and its dependence on ν and $\bar{\nu}$.
 - Calculation of nuclear shadowing requires the amplitudes $a^{(I,C)}$ for the states with different C -parity (e.g. F_2 and F_3) and isospin I (important for the calculation of the neutron excess correction).
- DIS sum rules for nuclei (Adler and GLS) help to fix unknown amplitudes.

The Adler sum rule (ASR)

$$S_A = \int_0^{M_A/M} dx (F_2^{\bar{\nu}} - F_2^{\nu}) / (2x) = 2I_z$$

- In parton model S_A is the difference between number of valence u and d in the target. $S_A(p) = +1$, $S_A(n) = -1$.
- ASR is independent of Q^2 and survives strong interaction effects because of CVC.
- Nonconservation of axial current is neglected in derivation of ASR. Therefore, ASR is good for sufficiently high Q^2 but violated at low Q^2 (e.g. by PCAC terms).
- For generic nucleus of Z protons and N neutrons $S_A/A = \beta = (Z - N)/A$.

Explicit calculation of different nuclear effects:

- FMB correction cancels out. So does the pion correction.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between these corrections fixes $\text{Re } a^{(1,-)} / \text{Im } a^{(1,-)}$ and the strength of the NS correction in the isovector ($I = 1, C = -1$) channel.

Gross–Llewellyn-Smith sum rule (GLS)

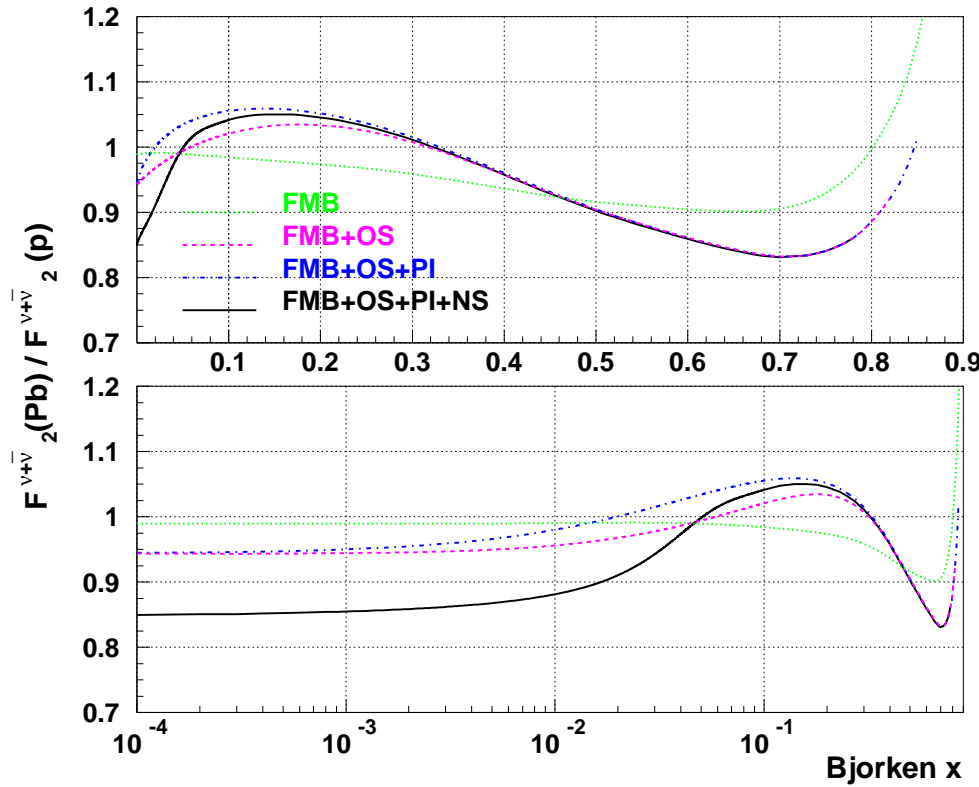
$$S_{\text{GLS}} = \frac{1}{2} \int_0^{M_A/M} dx (F_3^{\bar{\nu}} + F_3^{\nu})$$

- In parton model $S_{\text{GLS}} = 3$ is the number of valence u and d quarks in the target.
- In QCD the relation between S_{GLS} and the baryon number only holds at asymptotic Q^2 . S_{GLS} is affected by QCD radiative corrections and HT effects $S_{\text{GLS}} = 3(1 - \alpha_S/\pi - 3.25(\alpha_S/\pi)^2 + \dots) + \text{HT}_{\text{GLS}}$.

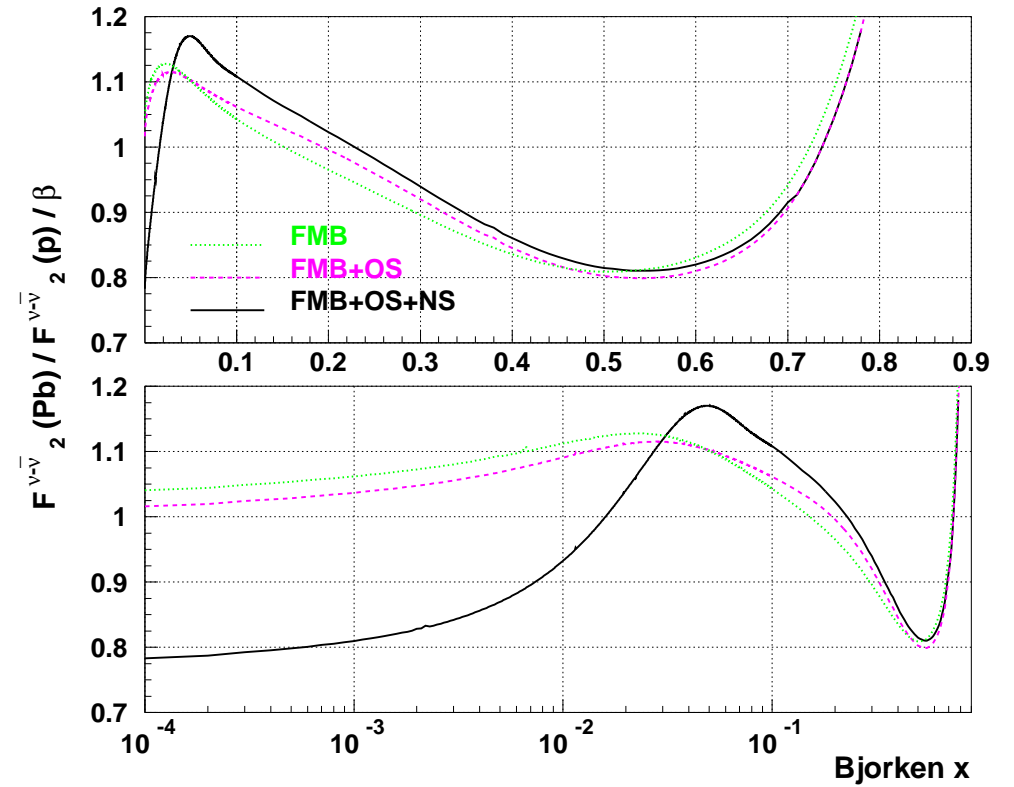
Explicit calculation of different nuclear effects:

- FMB correction to S_{GLS} cancels out. Nuclear pion correction to $F_3^{\nu+\bar{\nu}}$ vanishes.
- Both off-shell (OS) and nuclear shadowing (NS) corrections are nonzero. The requirement of exact cancellation between these corrections fixes $\text{Re } a^{(0,-)} / \text{Im } a^{(0,-)}$ and the strength of the NS correction in the isoscalar ($I = 0, C = -1$) channel.

Dependence of Nuclear Effects on C -parity

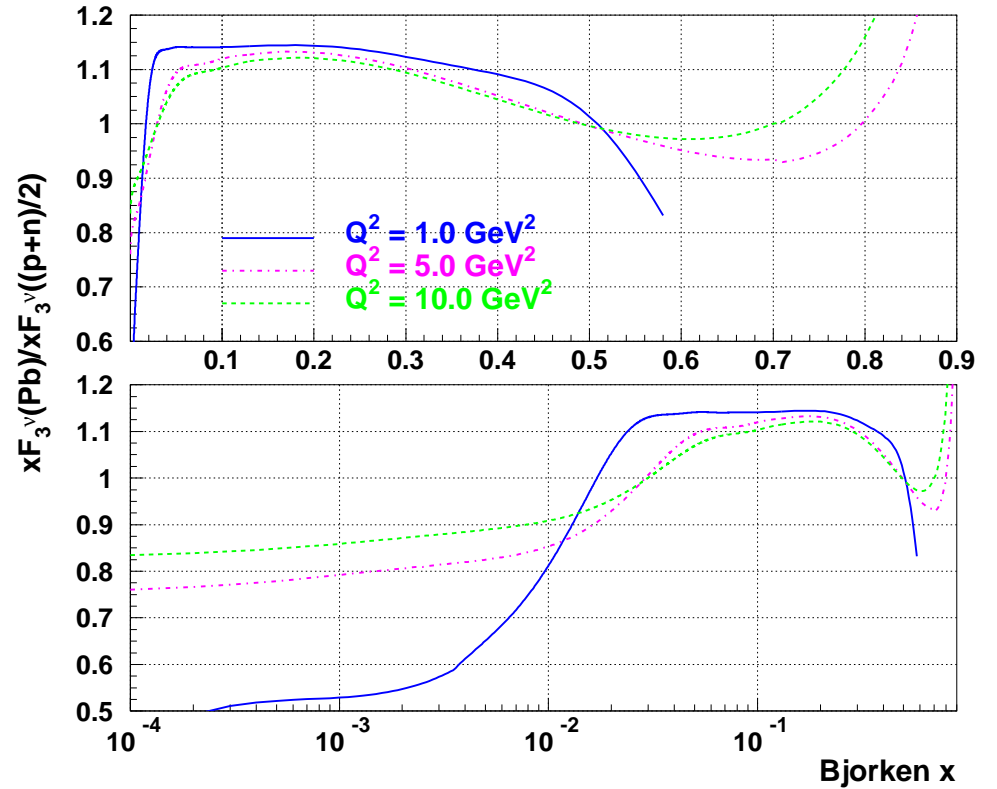
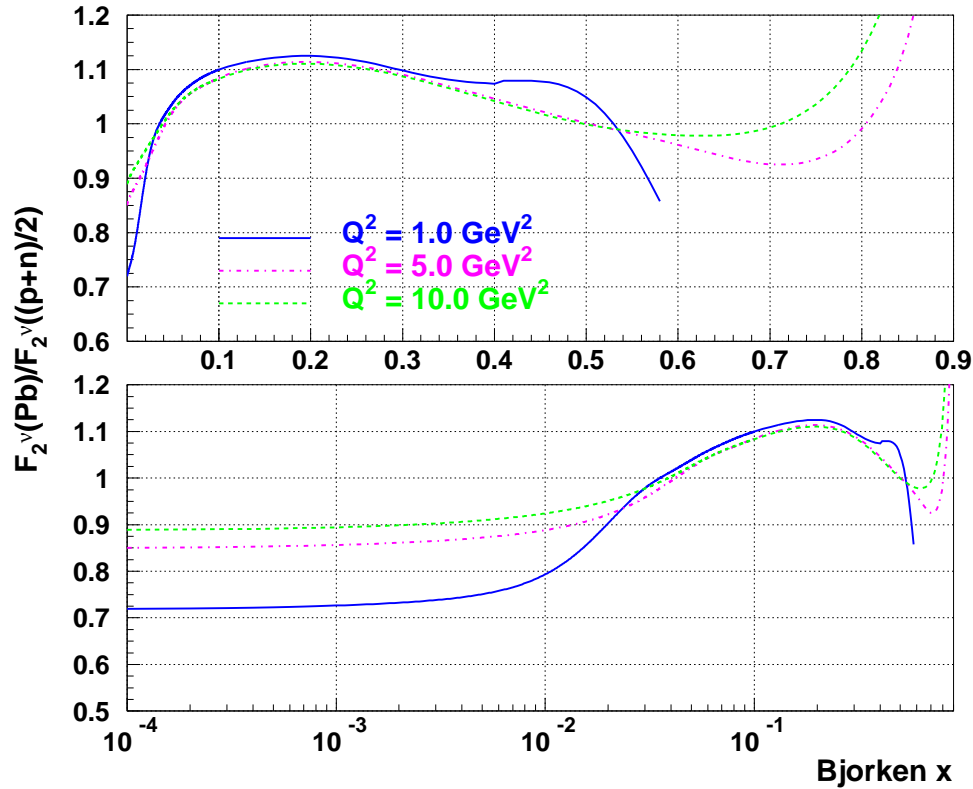


The ratio $\frac{1}{A} F_2^{(\nu+\bar{\nu})A} / F_2^{(\nu+\bar{\nu})p}$ calculated for ^{207}Pb at $Q^2 = 5 \text{ GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS), nuclear pion excess (PI) and coherent nuclear processes (NS).



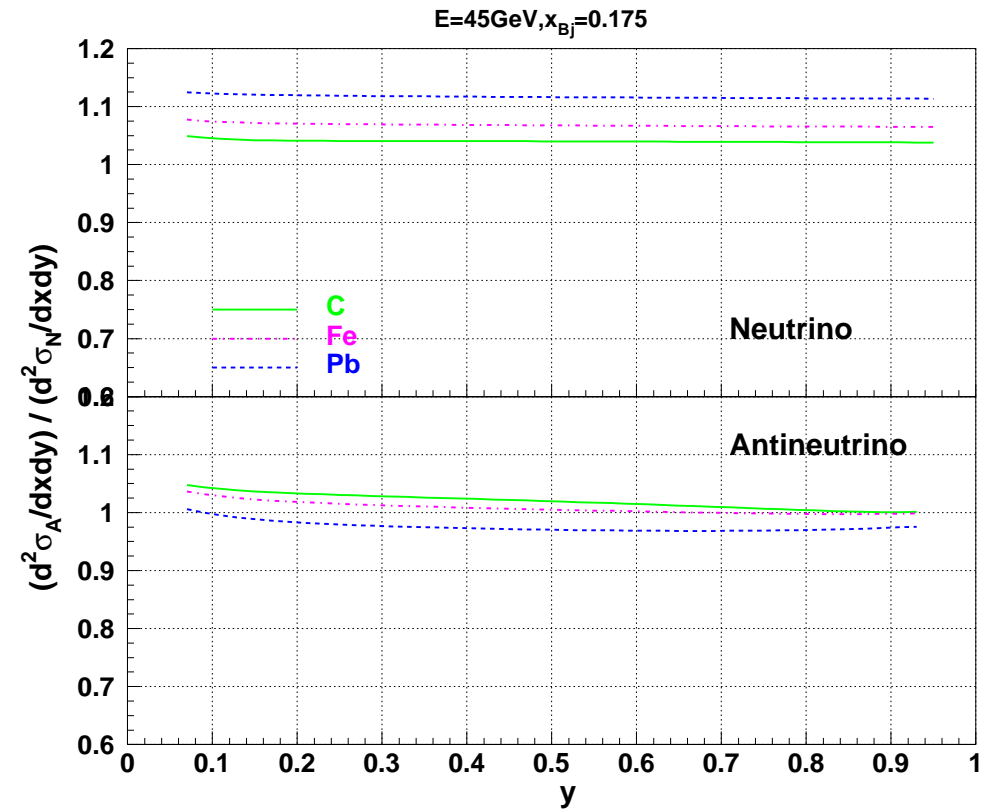
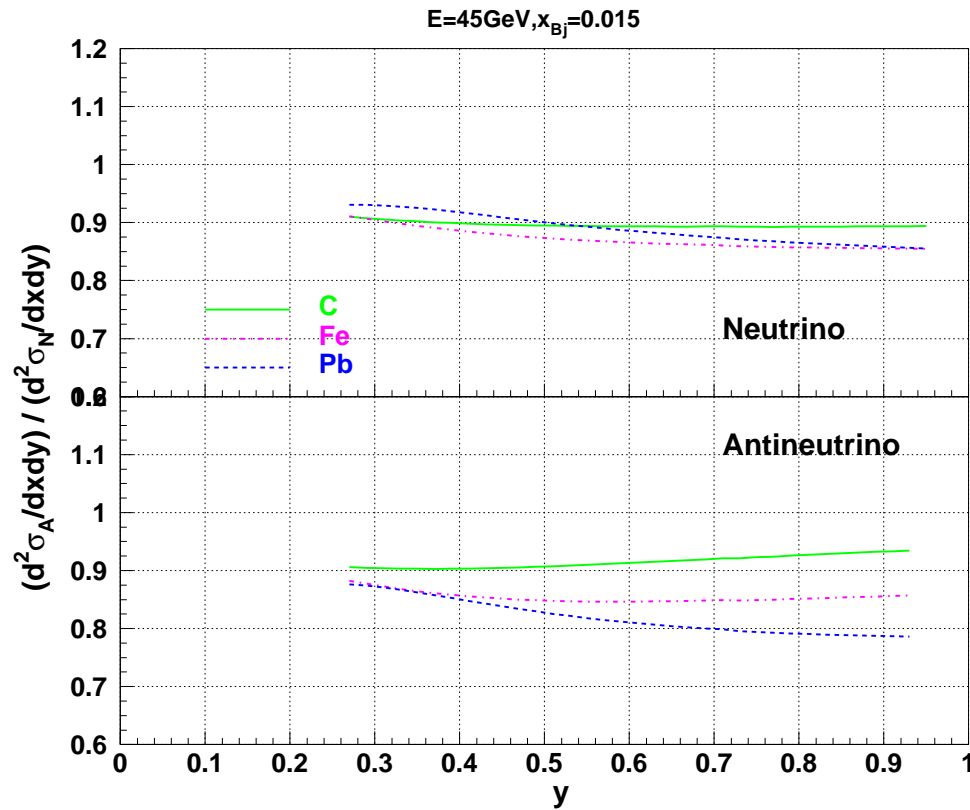
The ratio $\frac{1}{A} F_2^{(\nu-\bar{\nu})A} / (\beta F_2^{(\nu-\bar{\nu})p})$ calculated for ^{207}Pb at $Q^2 = 5 \text{ GeV}^2$. The labels on the curves correspond to effects due to Fermi motion and nuclear binding (FMB), off-shell correction (OS) and coherent nuclear processes (NS).

Nuclear Effects for F_2^ν and F_3^ν

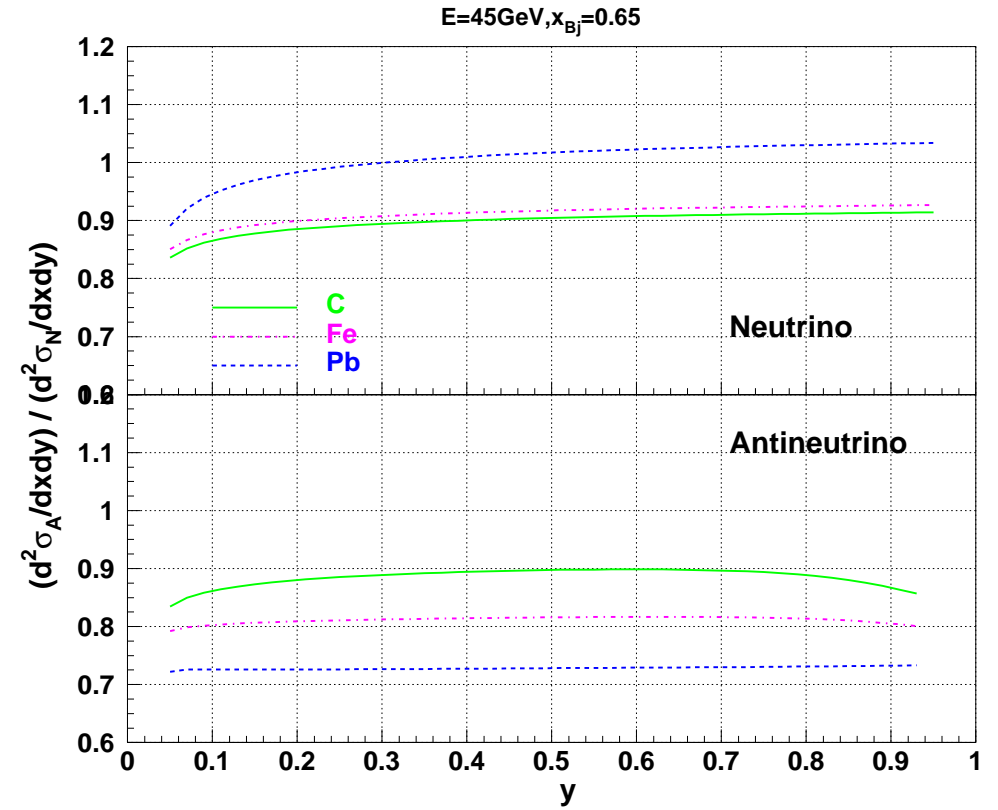
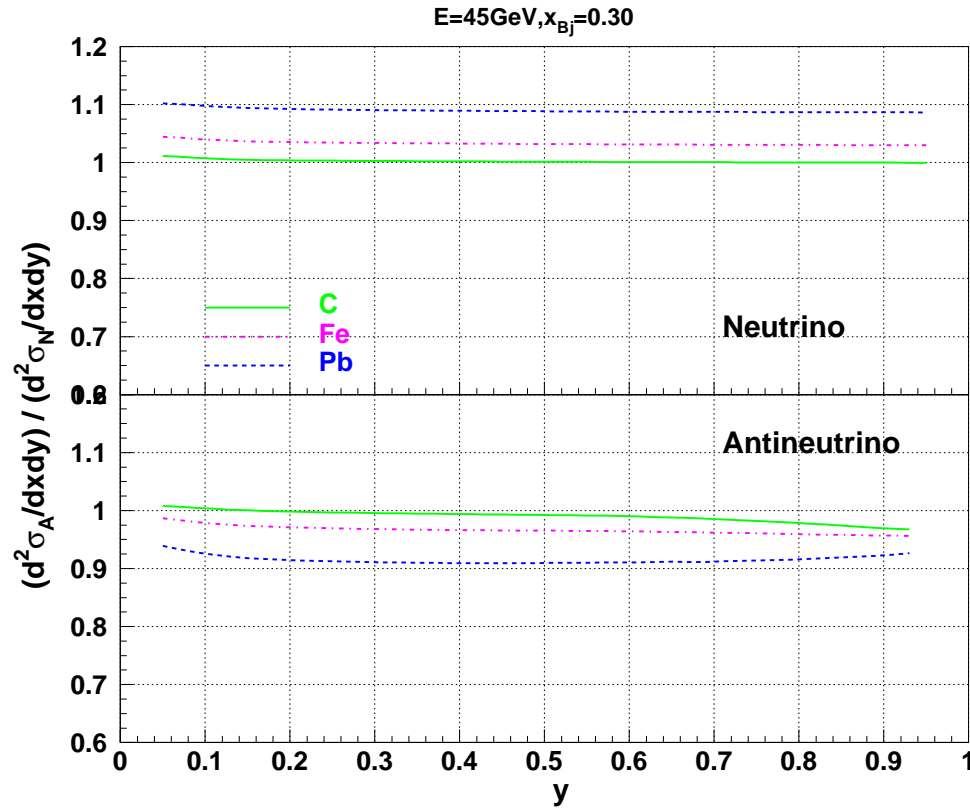


The ratios of CC neutrino structure functions for ^{207}Pb normalized to one nucleon and those of the isoscalar nucleon $(p+n)/2$ (left panel for F_2 and right panel for xF_3).

(Anti)neutrino cross sections

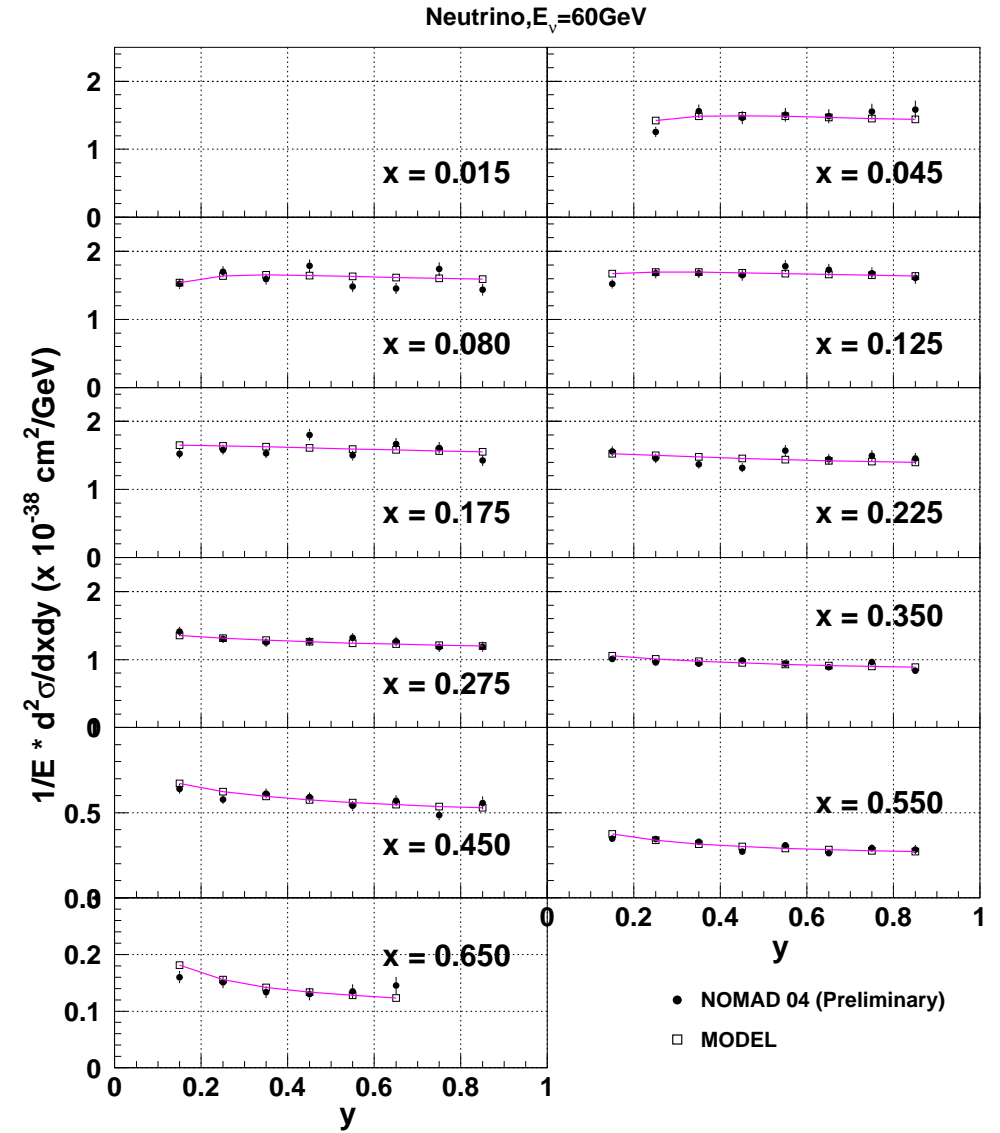
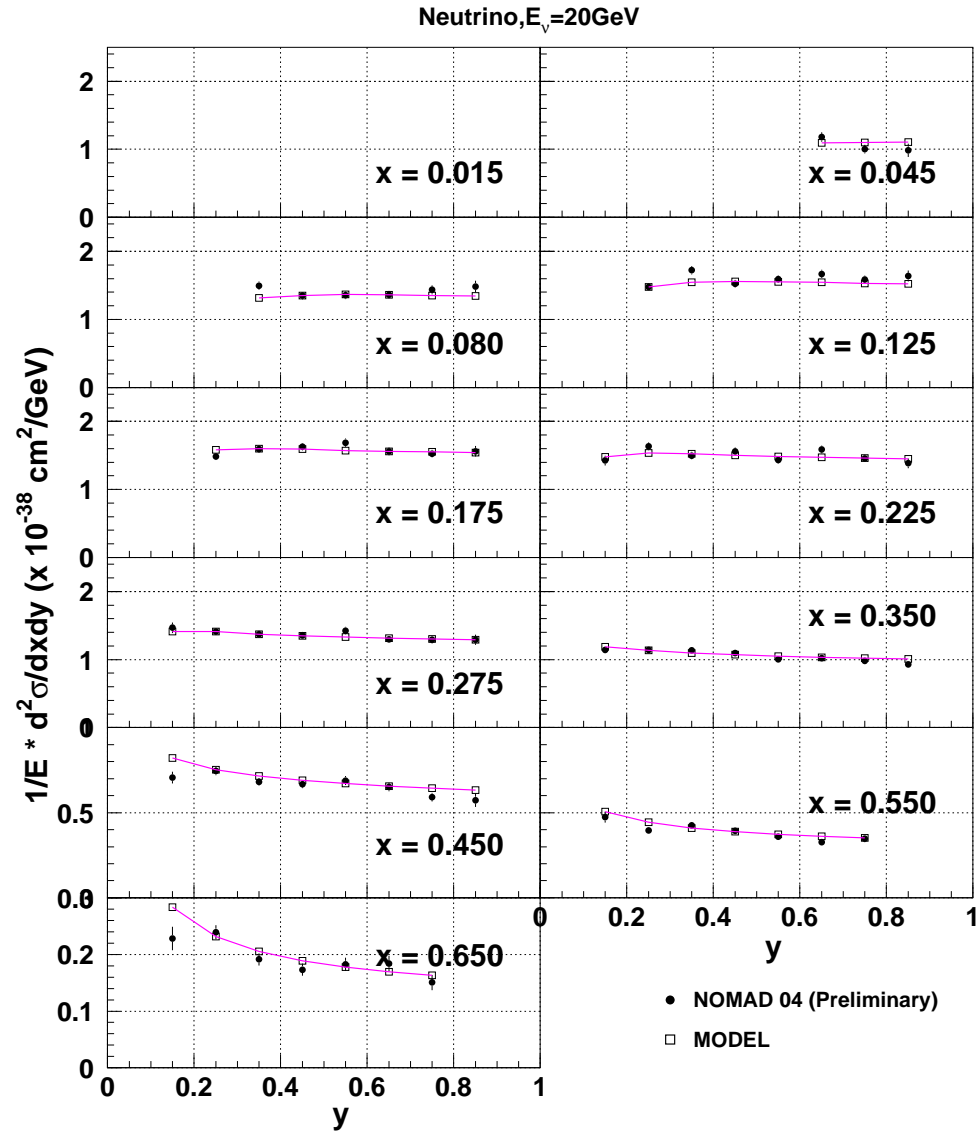


Note the cancellation of nuclear effects for antineutrino at $x \sim 0.1 \div 0.2$. Antineutrino cross section is not very sensitive to the target material in this region.



Note the cancellation of nuclear effects at $x \sim 0.3$ for the isoscalar target (^{12}C). Nuclear effects in this region are driven by the neutron excess.

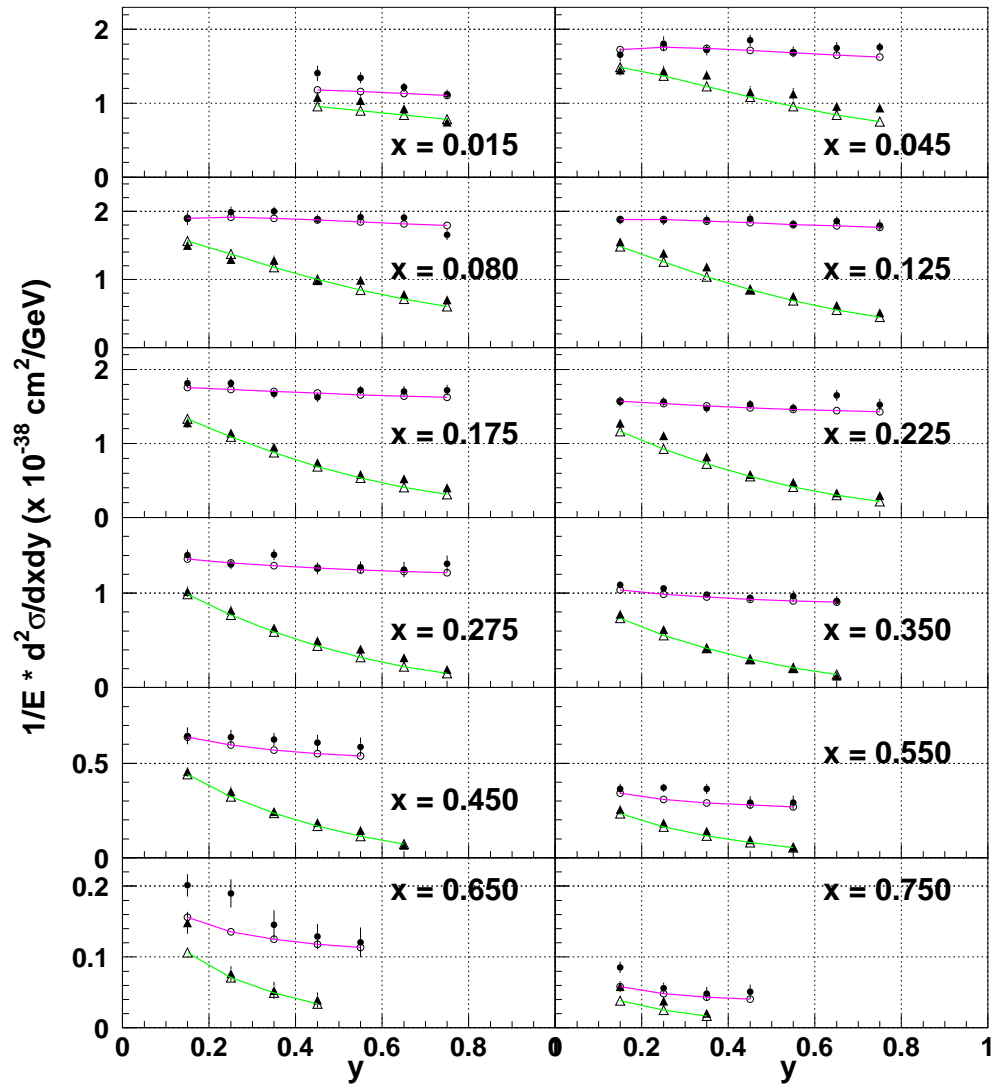
Comparison with NOMAD νC cross sections



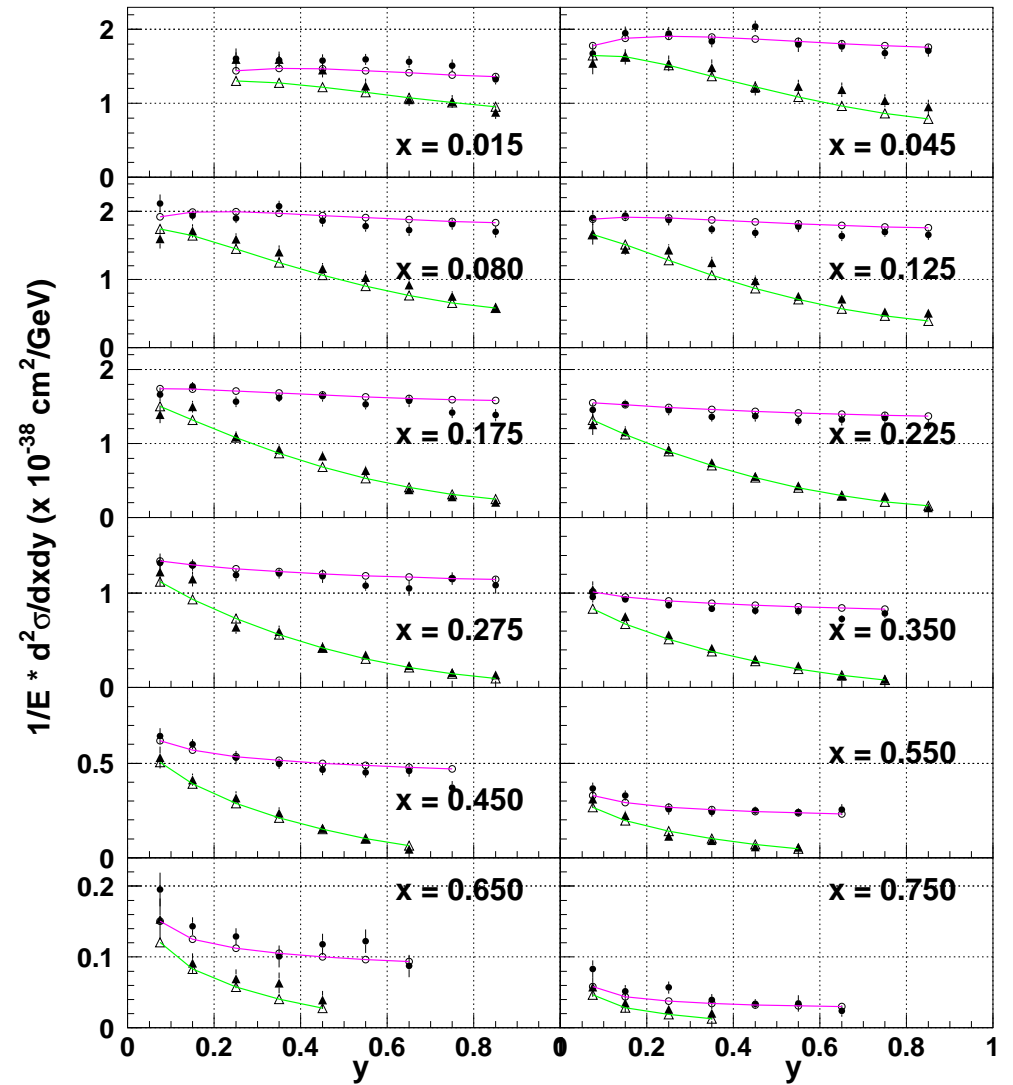
Data: R. Petti hep-ex/0602022

Comparison with NuTeV $\nu(\bar{\nu})$ Fe cross sections

E=85GeV

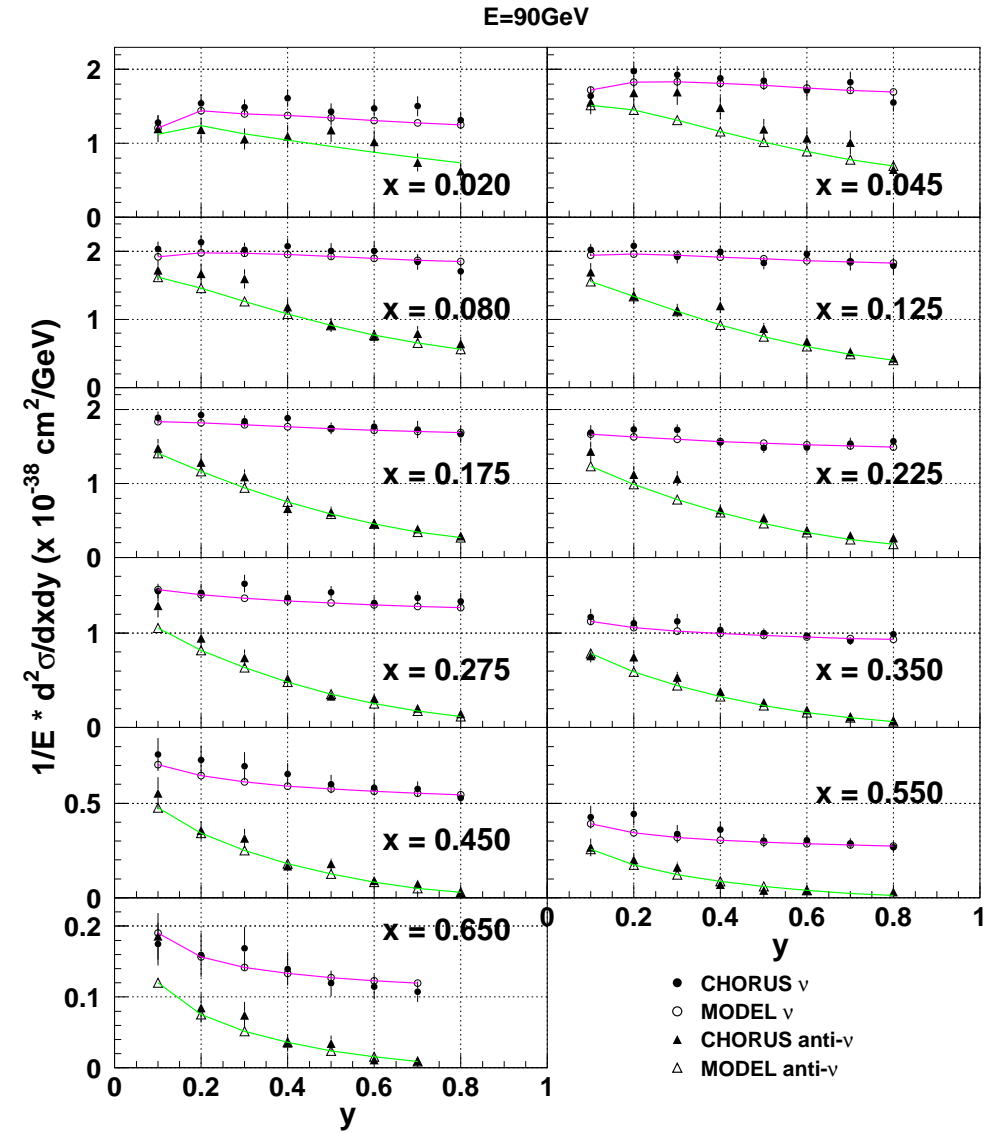
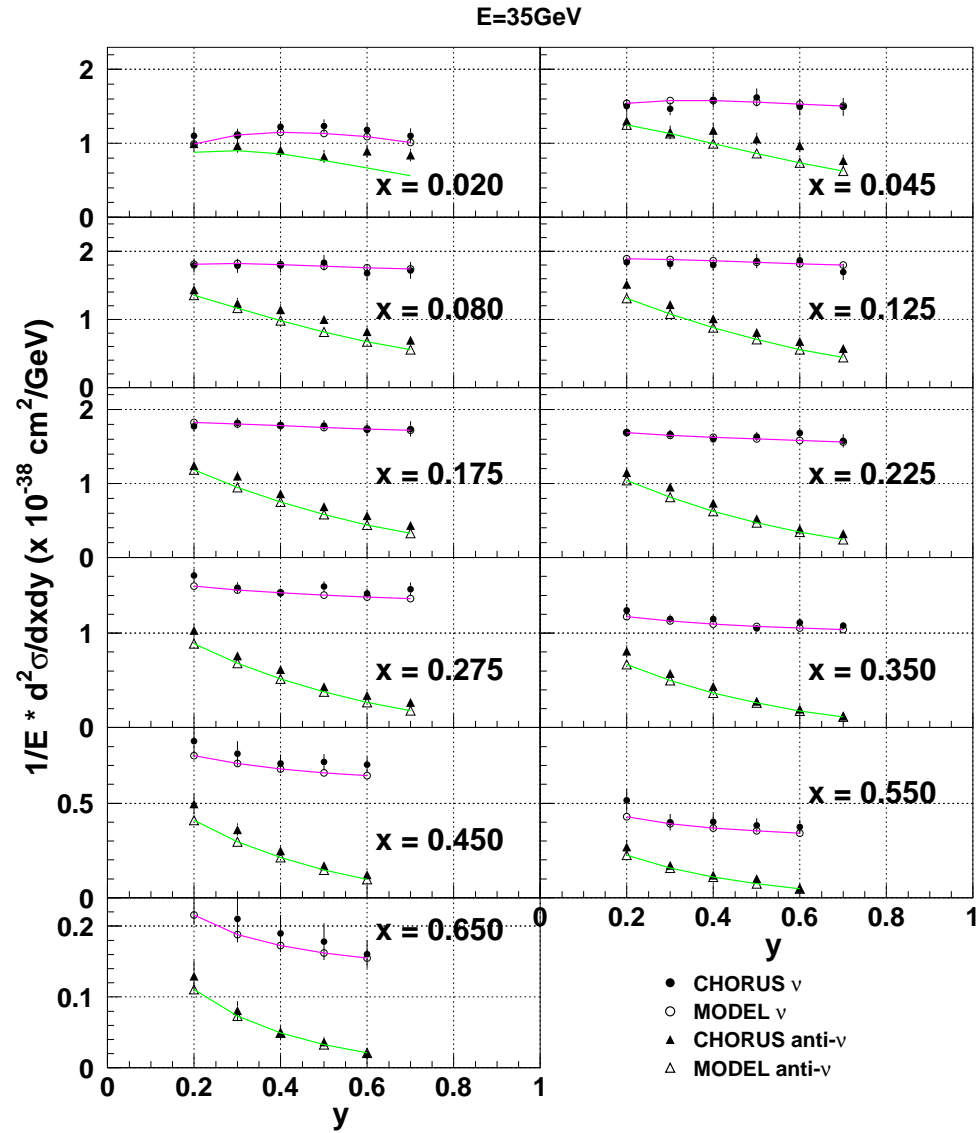


E=170GeV



Data: M. Tzanov et.al. hep-ex/0507040

Comparison with CHORUS $\nu(\bar{\nu})$ Pb cross sections



Data: CHORUS Collab. PLB632(2006)65

Summary

- A detailed quantitative study of nuclear EMC effect was performed in a wide kinematical region of x and Q^2 and for nuclei from ^4He to ^{207}Pb . A model was developed which takes into account the QCD treatment of the nucleon structure functions and addresses a number of nuclear effects including nuclear shadowing, Fermi motion and nuclear binding, nuclear pions and off-shell corrections to bound nucleon structure functions.
- The approach was applied to calculate neutrino-nuclear DIS structure functions and cross sections. We discussed in detail how the nuclear effects depend on the structure function type and C -parity.
- Our predictions for (anti)neutrino inelastic differential cross sections agree well with recent data on ^{12}C (NOMAD), ^{56}Fe (NuTeV), ^{207}Pb (CHORUS).